Definition of $U(\mathfrak{g}, e)$

$\mathfrak{g} = \bigoplus_{j \in \mathbb{Z}} \mathfrak{g}(j)$, where $\mathfrak{g}(j) = \{x \in \mathfrak{g} \mid [h, x] = jx\}$.
Let $\chi \in \mathfrak{g}^*$ be *dual* to e, with respect to a nondegenerate symmetric invariant bilinear form on $\mathfrak{g}$.

For $x, y \in \mathfrak{g}(-1)$, define $\langle x, y \rangle = \chi([x, y])$.
$\langle \cdot\, , \cdot \rangle$ is a nondegenerate skew-symmetric bilinear form on $\mathfrak{g}(-1)$.
Let $\mathfrak{k}$ be a *Lagrangian subspace* of $\mathfrak{g}(-1)$.

Let $\mathfrak{m} = \mathfrak{k} \oplus \bigoplus_{j \leq -2} \mathfrak{g}(j)$.
Then χ restricts to a *character* of $\mathfrak{m}$.
Write $\mathbb{C}_\chi$ for the corresponding 1-dimensional $U(\mathfrak{m})$-module.

$$Q_\chi = U(\mathfrak{g}) \otimes_{U(\mathfrak{m})} \mathbb{C}_\chi.$$

$$U(\mathfrak{g}, e) = \mathrm{End}(Q_\chi)^{\mathrm{op}}.$$

PBW theorem and Skryabin's equivalence

$x \in \mathfrak{g}(j)$ has Kazhdan degree $j+2$.
Gives the Kazhdan filtration on $U(\mathfrak{g}, e)$.

Theorem (Premet/Gan–Ginzburg '02)

$$\operatorname{gr} U(\mathfrak{g}, e) \cong \mathbb{C}[e + \mathfrak{g}^f] \cong S(\mathfrak{g}^e).$$

$\mathrm{Wh}(\mathfrak{g}, \mathfrak{m}_\chi)$ is the full subcategory of $U(\mathfrak{g})$-mod of modules on which such that $x - \chi(x)$ acts locally nilpotently for all $x \in \mathfrak{m}$.

Theorem (Skryabin '02)

The functor

$$Q_\chi \otimes_{U(\mathfrak{g},e)} ? : U(\mathfrak{g}, e)\text{-}mod \to \mathrm{Wh}(\mathfrak{g}, e)$$

is an equivalence of categories.

A theorem

$\mathfrak{g} = \mathfrak{sp}_{2n}$, all parts of $\mathbf{p}$ occur with even multiplicity.

$A \in \mathrm{sTab}(F)$ and $z \in \mathbb{C}$

A_z is the subtable of A of boxes with entries in $z + \mathbb{Z}$.

$A_{\pm z} = A_z \cup A_{-z}$

$\ell(A_*, i)$ is the length of row i in A_*.

We define $\mathrm{sTab}^\diamond(F) \subseteq sTab(F)$ by saying $A \in \mathrm{sTab}^\diamond(F)$ if:

- A_z is row equivalent to column strict for all z;
- $\ell(A_z, i-1) \leq \ell(A_z, i)$ for all z and i;
- $|\ell(A_z, i) - \ell(A_{-z}, i)| \leq 1$ for all z and i;
- if $\ell(A, i)$ is odd, then $\ell(A_{\pm z}, i)$ is even for all $z \in \mathbb{C} \setminus \mathbb{Z}$;
- if $\ell(A, i)$ is even, then $\ell(A_{\pm z}, i)$ is odd for at most one $z + \mathbb{Z} \neq \mathbb{Z}$, and if $z \in \mathbb{C} \setminus \frac{1}{2}$, then $\ell(A_{\pm z}, i-1) < \ell(A_{\pm z}, i)$.

Theorem

$L(A, \mathfrak{q})$ is finite dimensional if and only if A is C-conjugate to an element of $\mathrm{sTab}^\diamond(F)$.