Spontaneous Partial Supersymmetry Breaking in $\mathcal{N}=2$ Supergravity

(Applications of the Embedding Tensor)

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with Vicente Cortés, Bobby Gunara, Christoph Horst, Jan Louis and Hagen Triendl
Why $N=2$ supersymmetry?

Compactifications of string theory often lead to $N=2$ supergravity in 4D with many massless scalar fields/moduli.

$N=2$ supersymmetric field theories are very constrained with many relations between couplings which make them easier to study, but there are no chiral fermions.

A Standard Model extension with an $N=2$ gauge sector has very different signals compared to the MSSM Fox et al `02, Herquet et al

$N=2$ Dirac fermions as dark matter candidates De Simone et al, Goodsell et al

Can $N=2$ supersymmetry be spontaneously broken to $N=1$ (in the visible sector) with a low-energy chiral spectrum? Antoniadis et al, Polonsky et al

We have looked at spontaneous supersymmetry breaking in supergravity
Partial supersymmetry breaking in supergravity

There is a no-go theorem which rules out spontaneous $N=2$ to $N=1$ supersymmetry breaking in Minkowski vacua for a large class of supergravities Cecotti, Girardello & Porrati `84
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10 years later a few counter-examples were produced using special choices of matter content Ferrara et al `95

We used the embedding tensor formalism to show that one can find an $N=1$ vacuum in any Abelian $N=2$ supergravity with two commuting isometries Louis, Smyth & Triendl `09

Furthermore, a given $N=2$ supergravity can have both $N=1$ & $N=2$ vacua inside the Kähler cone LST `12
The embedding tensor formalism \textit{de Wit, Samtleben & Trigiante '05}

The embedding tensor $\Theta_\Lambda^\lambda$ is a spurionic matrix of charges, which selects the subset of the global symmetry $t_\lambda$ generators that get used in a gauging:

\[
Y_\Lambda = \Theta_\Lambda^\lambda t_\lambda \\
\Theta_\Lambda^\lambda = (\Theta_I^\lambda, -\Theta^{I\lambda})
\]

\[
\delta_\alpha q^u = \alpha^\Lambda \Theta_\Lambda^\lambda k_\lambda^u(q)
\]

\text{electric charges} \quad \text{magnetic charges} \quad \text{Killing vectors}
The embedding tensor formalism de Wit, Samtleben & Trigiante `05

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**electric charges**

$$\delta_\alpha q^u = \alpha^\Lambda \Theta^\lambda_\Lambda k^u_\lambda(q),$$

**magnetic charges**

We can then define the covariant derivative as

$$D_\mu q^u = \partial_\mu q^u - A_\mu^I \Theta^\lambda_\Lambda k^u_\lambda - A_{\mu I} \Theta^{I\lambda} k^u_\lambda$$

and we enforce a locality constraint such that one can always take a symplectic rotation to a frame with only electric charges
The embedding tensor formalism applied

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Fix the physics and then solve for the theory

Possible as the embedding tensor $\Theta_{\Lambda}^\lambda$ is spurionic i.e. it transforms
To find an $N=1$ supersymmetric vacuum of $N=2$ supergravity we can solve the Killing spinor equations. For Minkowski vacua these simplify to just the scalar parts (additional indices suppressed):

$$\delta \Psi = \cdots + S_A(\Theta^\lambda, q^u, k_\lambda) \cdot \epsilon^A$$

The Killing spinor equations for an $N=1$ vacuum are then

**unbroken susy** \( S_1(\Theta^\lambda, q^u, k_\lambda) \cdot \epsilon^1 = 0 \)

**broken susy** \( S_2(\Theta^\lambda, q^u, k_\lambda) \cdot \epsilon^2 \neq 0 \)
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The Killing spinor equations for an $N=1$ vacuum are then

\[ S^\infty \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ S_1(\Theta^\lambda_\Lambda, q^u, k_\lambda) \cdot \epsilon^1 = 0 \]

\[ S_2(\Theta^\lambda_\Lambda, q^u, k_\lambda) \cdot \epsilon^2 \neq 0 \]
To find an $N=1$ supersymmetric vacuum of $N=2$ supergravity we can solve the Killing spinor equations. For Minkowski vacua these simplify to just the scalar parts (additional indices suppressed):

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\begin{align*}
\text{unbroken susy} & \quad \Rightarrow \quad S_1(\Theta^\lambda_A, q^u, k^\lambda) \cdot \epsilon^1 = 0 \\
\text{broken susy} & \quad \Rightarrow \quad S_2(\Theta^\lambda_A, q^u, k^\lambda) \cdot \epsilon^2 \neq 0
\end{align*}

We solve these equations to find the $\Theta^\lambda_A$, i.e. the gaugings or charges, that give rise to an $N=1$ vacuum
For an $N=1$ vacuum of $N=2$ the embedding tensor components are fixed by the Killing vectors and prepotential $\mathcal{F}$ at the $N=1$ point

$$\Theta^{\lambda}_{\Lambda}(k_{\lambda}, \mathcal{F})|_{N=1}$$

The $N=1$ condition can be solved if there are two commuting isometries
Properties of the general solution LST ’09 - `12, with Horst & Gunara 12

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The $N=1$ condition can be solved if there are two commuting isometries

◆ There is no constraint on the vector multiplets

◆ Protected from quantum corrections

◆ The required gaugings have a string interpretation - non-geometric flux

◆ We extended this result to all special quaternionic-Kähler manifolds (c-map)

◆ Both $N=1$ and $N=2$ vacua can exist inside the Kähler cone LST '12
Further surprises

We also constructed the low-energy effective supergravity in the $N=1$ vacuum

- A very large number of massless fields become **massive** - moduli stabilisation
  - for cubic $\mathcal{F}'s$ all vector multiplet scalars and $2n+2$ of $4n$ hyperscalars fixed

- A new construction of Kähler quotients (of large dimension) of quaternionic-Kähler manifolds  LST & Cortes `11

- Found that special quaternionic-Kähler manifolds have a natural, global complex structure

Furthermore,

- Easily extended to rigid spontaneous $N=2$ to $N=1$ breaking, although the limit between local and global is tricky  LST & Gunara `12 (c.f. Antoniadis et al `12)

- No $N=2$ AdS vacua in $N=2$ supergravity ($d=4, 5$ or $6$) from $SU(3) \times SU(3)$ compactifications of Type II string theory and M-theory. LST `12
The embedding tensor formalism has useful applications, beyond constructing the most general supergravities. Choose some interesting physics and then solve for the embedding tensor components that allow for this i.e. the charges and gaugings. $N=2$ to $N=1$ spontaneous supersymmetry breaking in supergravity is fine and robust - nothing special is required. The $N=1$ low-energy effective action is not generic and many massless scalars can get masses. New construction of Kähler quotients of quat.-Kähler manifolds. No $N=2$ AdS vacua in $N=2$ in d=4, 5 and 6 from Type II and M-theory.