Fluxes, Warping and Constraints in F-theory

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Chirality + Flux: 1111.1232 [hep-th] with H. Hayashi
Warping + Flux: 1202.0285 [hep-th] with D. Klevers, M. Poretschkin
Constraints: 1204.3092 [hep-th] with W. Taylor
Introduction and Motivation
Systematics of compactifications

- Fundamental theory (String/M-/F-theory)

  low energies  Wilsonian effective action: integrate out massive states which are heavier than a certain energy scale

- Classifying the core data of the effective actions by discrete information:

  \[ \vec{N} = \begin{pmatrix} \vec{N}_{\text{geom}} \\ \vec{N}_{\text{brane}} \\ \vec{N}_{\text{flux}} \end{pmatrix} \]

  - topological data of the compactifications manifold (Hodge-numbers, intersection numbers, Chern classes)
  - topological data of the brane configuration (number of branes, wrapping numbers, intersections...)
  - flux data, bulk + brane

- field dynamics is encoded by effective action effective potential, vacua, ... phenomenology
Looking for the unifying setup...

- ideally: look for unifying fundamental setup where Type IIA/IIB and the various branes are just different aspects

  \[ \Rightarrow \text{M-theory in 11d} \]

  **key example:** D6-branes admit geom. interpretation in M-theory \( \Rightarrow \) unify \( \vec{N}_{\text{geom}} \) and \( \vec{N}_{\text{brane}} \)

- major drawback: \( \mathbb{M}^p \times X_{11-p} \) effective theory in even dim.
  \( \Rightarrow \) internal manifold is odd dim.

**F-theory** provides an ideal setup for:

(1) unifying 7-brane and bulk physics \( \vec{N}_{\text{brane}} \) & \( \vec{N}_{\text{geom}} \) in complex geometries

(2) promising phenomenological scenarios (GUTs, moduli stabilization)
Obstacles in the study of F-theory

- contrast to string/M-theory: no 12-dimensional F-theory effective action
also: fundamental formulation is poorly understood

- F-theory physics is often studied using limits and dualities:
  - weak coupling limit with D7-branes and O7-planes
  - F-theory / heterotic duality
  - local geometries

Only known way to extract generic features of F-theory effective actions is via its formulation as a limit of M-theory.

- **Remark:** if objects like G\(_4\) - flux and M5-branes are used in the context of F-theory this limit is always understood
Goals of this talk:

- **Part I:** Derive 4D chiral index for F-theory compactifications using M-theory $G_4$-flux in the F-theory limit

- **Part II:** Derive corrections to the F-theory gauge coupling function due to flux using M-theory warping

- **Part III:** Comment on constraints imposed by F-theory

**Message:**
- F-theory effective action can be reliably studied: bulk + 7-brane physics in a unified $N=1$ framework
- M-theory origin of various F-theory effects can be unexpected
F-theory via M-theory
F-theory compactifications

- Type IIB has non-perturbative $Sl(2, \mathbb{Z})$ symmetry rotating $\tau = C_0 + ie^{-\phi}$
  ⇒ interpret $\tau$ as complex structure of a two-torus (2 auxiliary dimensions)

- minimally supersymmetric F-theory compactifications:
  › F-theory on torus fibered Calabi-Yau 4-fold $Y_4$
    ⇒ 4 dim, N=1 supergravity theory
    ⇒ base $B_3$ is a Kähler manifold

- singularities of the fibration are crucial to encode 7-brane physics
  ⇒ pinching torus indicates presence of 7-branes magn. charged under $\tau$

- brane and bulk physics encoded by complex geometry

[Vafa] [Morrison, Vafa]
F-theory via M-theory

- F-theory viewed as auxiliary ‘12 dim.’ theory (torus volume unphysical)

- F-theory effective actions has to be studied via M-theory
  Consider M-theory on space $T^2 \times M_9$

  $\tau$ is the complex structure modulus of the $T^2$, $v$ volume of $T^2$

F-theory limit:

(1) **A-cycle**: if small than M-theory becomes Type IIA
(2) **B-cycle**: T-duality $\Rightarrow$ Type IIA becomes Type IIB, $\tau$ is indeed dilaton-axion
(3) grow extra dimension: send $v \to 0$ than T-dual B-cycle becomes large

- can be generalized for singular $T^2$ fibrations: e.g. Taub-NUT $\to$ D6 $\to$ D7
F-theory / M-theory geometries

- F-theory geometries can be constructed and analyzed
  - singularities of elliptic fibration induce non-Abelian gauge symmetry
  - singularity resolution: (resolution at each codimension)

- **Examples:** compact, fully resolved Calabi-Yau three-/fourfolds
  ⇒ toric geometry

- **Unification of** $\mathbf{N}_{\text{geom}}$ and $\mathbf{N}_{\text{brane}}$ on resolved Calabi-Yau manifolds
  ⇒ bulk and 7-brane geometries systematically classified by smooth higher-dimensional complex geometries!
M-theory on resolved CY manifolds

- physical interpretation of resolution only possible in M-theory
  - moving branes apart on the B-circle:
    - Coulomb branch of the lower-dimensional gauge theory:
      \[ G \rightarrow U(1)^{\text{rank } G} \]

- Massive states from M2 branes on geometric 2-cycles:
  - M2-branes on resolution \( \mathbb{P}^1 \)'s over generic points of \( S \) ⇒ massive ‘W-bosons’ of \( G \)-breaking
  - M2-branes on resolution \( \mathbb{P}^1 \)'s over intersection ⇒ massive matter multiplets
  - M2-branes on the elliptic fiber ⇒ massive Kaluza-Klein modes

- All massive states have to be integrated out to determine Wilsonian effective action ⇒ in circle compactification also KK-modes are crucial!!
4D F-theory effective actions via M-theory

- Effective actions can be computed via M-theory / 11-dimensional supergravity on the resolved Calabi-Yau fourfolds.

F-theory on singular CY\(_4\), ‘Gauge bundle’

- 4d, N=1 effective theory with non-Abelian gauge symmetry G
- 3d, N=2 effective theory pushed to 3d Coulomb branch: \(U(1)^{\text{rk}G}\)

M-theory on resolved \(\tilde{\text{CY}}\)_\(_4\), \(G\)_4-flux

- 3d, N=2 effective theory with only abelian gauge symmetries

- Explicit: N=1 characteristic data determining the action

Compare

4d / 3d

[TG] [TG, Kerstan, Palti, Weigand]
[TG, Savelli] [TG, Hayashi]
[TG, Klevers, Poretschkin]
Part I: Chiral index from flux
Finding F-theory fluxes for fibered fourfolds

- Specification of flux bundle in F-theory is hard
  spectral cover-type methods, link to weak coupling
  → Schäfer-Nameki’s talk
  → Weigand’s talk

- M-theory dual model on fully resolved $\widetilde{CY}_4$:
  non-Abelian groups: new resolution two-forms (dual to resolution divisors)

\[
\omega_i \quad i = 1, \ldots, \text{rank}(G)
\]

fully resolved geometry has new intersection numbers (compute explicitly):
\[
\int \omega_i \wedge \omega_j \wedge \omega_k \wedge \omega_l
\]

extra $U(1)$'s: new resolution two-forms $\tilde{\omega}_m \quad m = 1, \ldots, n_{U(1)}$

- M-theory fluxes we consider:

\[
G_4 = m^{\Sigma \Lambda} \omega_\Sigma \wedge \omega_\Lambda
\]

- $m^{\Sigma \Lambda}$ subject to vanishing conditions to lift to F-theory
- $m^{\Sigma \Lambda}$ subject to quantization conditions (Freed-Witten) [Collinucci,Savelli]
  → Savelli’s talk
F-theory chiral spectrum via M-theory

- determination of charged chiral spectrum is much harder:
  - chirality induced by fluxes on 7-branes, but in M-theory on resolved 4-fold there are no chiral fields
  - Chirality formulas for M/F-theory setups?

\[
\chi(R) = \int_{S_R} G_4
\]

\[
G_4 = \langle dC_3 \rangle \quad \text{flux on resolved fourfold}
\]

- 3D one-loop Chern-Simons terms linked to 4D chiral index \( \chi(R) \)
- 3D M-theory Chern-Simons terms computes anomaly free chiral spectrum

[Braun, Collinucci, Valandro]
[Marsano, Schäfer-Nameki]
[Krause, Mayrhofer, Weigand]
[TG, Hayashi]
[Intriligator, Jockers, Mayr, Morrison, Plesser]
[Küntzler, Schäfer-Nameki]

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[Küntzler, Schäfer-Nameki]

Thursday, June 28, 2012
3D Chern-Simons vs. 4D chiral matter

- 3d Chern-Simons terms: \( S_{CS}^{(3)} = \int \Theta_{ij} A^i \wedge F^j \quad U(1)^{\text{rk}G} \)

  - No such terms from classical circle reduction of 4d, N=1 theory ⇒ generated at one loop by massive fermions

    \[ \Theta_{ij}^{1-\text{loop}} = \frac{1}{2} \sum_{\text{rep}} n_{\text{rep}} \sum_{\lambda \in W(\text{rep})} \lambda_i \lambda_j \text{sign}(w_k \xi^i) \]

    [Aharony, Hanany, Intriligator Seiberg, Strassler]

  - M-theory on resolved \( \widetilde{CY}_4 \) has classical Chern-Simons term:

    \[ \Theta_{ij}^{\text{flux}} = \int_{\widetilde{CY}_4} G_4 \wedge \omega_i \wedge \omega_j \]

  - \( G_4 \) fluxes on \( \widetilde{CY}_4 \) count 4D chiral matter spectrum massive in the 3D Coulomb branch ⇒ determine chiral index

    \[ \Theta_{ij}^{1-\text{loop}} = \Theta_{ij}^{\text{flux}} \]

  - 4D anomaly induced by chiral matter is captured by 3D Chern-Simons term at one loop in the Coulomb branch (F-theory vs. M-theory)
Examples: with or without U(1)

- M-theory fluxes which do not break SU(5) gauge group
  consider e.g. base $\mathbb{P}^2 \times \mathbb{P}^1$
  (1) toric construction of fully resolved CY$_4$
  (2) computation of intersection numbers: fluxes of form $G_4 = m^{ij} \omega_i \wedge \omega_j$
  (3) computation of Mori cone to evaluate: $\text{sign} \left( \int_{\Sigma} J \right)$
    $\Rightarrow$ compute chirality by using one-loop equation

- U(1) restricted Tate model: simple way to globally obtain geometrically massless U(1) for a fourfold
  $\Rightarrow$ same program can be applied for fluxes of the form $G_4 = F_{U(1)} \wedge \omega_{U(1)}$

- Algorithmic implementation and model searches are possible!
  $\Rightarrow$ scanning over Kreuzer-Skarke list for fourfolds....
Part II: Warping and the gauge coupling function
Effective action of D7-branes with fluxes

- **study \( \alpha' \) corrections**: example - D7-brane fluxes \( F_{D7} \) are known to introduce corrections to effective 4D gauge coupling function:

consider stack of D7-branes on \( S \):

\[
(f_{D7})_{IJ} = \delta_{IJ} \int_S (J \wedge J + iC_4) + i\tau \int_S (F_{D7} \wedge F_{D7})_{IJ}
\]

for simplicity we will consider

\[
n^I \delta_{IJ} = \int_S (F_{D7} \wedge F_{D7})_{IJ}
\]

- generalization of these corrections in F-theory?

\[\Rightarrow\] need to find a derivation of these expressions in M-theory,

but: \( F_{D7}^4 \) would imply a four-derivative term in 11D SUGRA?
Back-reaction of fluxes

- Fluxes in M-theory compactifications on fourfolds induce warping:

\[ ds^2 = e^{-A(y)} \eta_{\mu \nu} dx^\mu dx^\nu + e^{A(y)/2} g_{ab} dy^a dy^b \]

e.o.m. and susy imply:

\[ \Delta_{Y_4} e^{-3A/2} = {}^* Y_4 (G_4 \wedge G_4) \]

\[ g_{ab} \text{ Calabi-Yau fourfold metric} \]

- solve warp-factor equation explicitly near 7-brane dual geometry

stack of D7-branes wrapping \( S^1 \) and \( U(N) \)

D6-branes are points on \( S^1 \)

\[ U(N) \rightarrow U(1)^N \]

periodic multi-centered Taub-NUT over \( S \)

M-theory lift

T-duality
Periodic multi-centered Taub-NUT

- local CY$_4$:

- Gibbons-Hawking metric:

\[
\begin{align*}
V &= 1 + \sum_{I=1}^{N} V_I \\
U &= \sum_{I=1}^{N} U_I \\
dV_I &= *_3 dU_I
\end{align*}
\]

\[
ds^2_{TN} = \frac{1}{V} (dt + U)^2 + V \, d\tau^2
\]

\[
\tau = \tau_0 + \frac{N}{2\pi i} \log(z/\Lambda)
\]

- Poisson resummation of an infinite line of Taub-NUT spaces with N centers

\[
V_I = \log(|z|) - \sum_{n>0} K_0(2\pi n |z|) \cos(2\pi n (x - x_I))
\]

[see also Ooguri,Vafa]

- leading: match metric with torus fibration

log-singularity near a D7-brane
Fluxes and solutions

- Include 7-brane fluxes on $TN_N^\infty \times S$:

\[ G_4 = F^{I}_{\text{flux}} \wedge \Omega_I \]

- Warp-factor equation can be solved analytically:

\[ \Omega_I = d\left(\frac{V}{V_I}(dt + U) - U_I\right) \]

- M-theory reduction including warping exactly gives flux-contribution to gauge coupling function + further $g_s$-corrections

3D kinetic term for vectors:

\[ G_{IJ} F^I \wedge *_3 F^J \quad \text{with} \quad G_{IJ} = \int_{Y_4} e^{3A/2} \Omega_I \wedge *_{Y_4} \Omega_J \]
Part III: Topological terms and constraints
Basic idea:

- in 6D F-theory compactifications on CY$_3$:
  - effective theory: strong constraints from anomalies (gauge+gravitational...)
  - F-theory geometry: topological properties of resolved elliptic fibrations can be matched with the anomaly constraints
    \[ \Rightarrow \text{relating terms in the effective action and spectrum} \text{ (Green-Schwarz)} \]

- in 4D F-theory compactifications on CY$_4$:
  - F-theory geometry: various topological properties genuine to consistent compactification
    \[ \Rightarrow \text{Can one relate terms in the effective action and spectrum despite the absence of certain anomalies? Are there universal constraints?} \]
4D topological terms

- to formulate constraints we also need topological terms - axion couplings
  - topological terms with 7-brane gauge field
    \[ \int b^\alpha \chi_\alpha \text{Tr}(F \wedge F) \rightarrow [S] = b^\alpha \omega_\alpha \text{ location of 7-branes in } B_3 \]
  - topological terms involving curvature
    \[ \int a^\alpha \chi_\alpha \text{tr}(R \wedge R) \rightarrow [K] = a^\alpha \omega_\alpha \text{ canonical class of } B_3 \]

→ our derivation using orientifold limit (likely also via M-theory as in 6D/5D)

- higher curvature terms are crucial to determine 7-brane configuration: discriminant:
  \[ [\Delta] = \text{rank}(G)[S] + [\Delta'] \]
  \( I_1 \) locus only visible via higher curvature terms in effective action
Constraints

- Relating the moduli spectrum to couplings: (no anomaly interpretation)

  define \( \langle x, y, z \rangle := K_{\alpha\beta\gamma} x^\alpha y^\beta z^\gamma \)

  - no gauge group
    - complex str. \( \text{Kähler} \)
    - Kähler + axion

      \[ 39 - 60 \langle a, a, a \rangle = C_{cs} + C_{sa} - C_{21} \]

    - including gauge group, but no charged matter

      \[ 39 - 60 \langle a, a, a \rangle = C_{cs} + C_{sa} - C_{21} + r_G + \frac{1}{6} r_G (c_G + 1) \langle a + b, a + b, b \rangle \]

- complications from a general 4D theory:

  - distinguish the moduli fields (all chiral multiplets in 4D), as in 5D
  - scalar potential e.g. due to fluxes \( \rightarrow \) focus on light fields
  - corrections and additional axion-(curvature)\(^2\), e.g. dilaton-axion at weak string coupling, additional het. axion

 derivation of Euler number of fourfold

[Sethi, Vafa, Witten]

[Andreas, Curio]
Conclusions

- **M-theory to F-theory limit is powerful (double-dimensional reduction)**
  - importance of **Chern-Simons terms**: study theory and formulate constraints
    loop corrections ↔ classical M-theory results (include Kaluza-Klein modes)
  - importance of **warping effects** in M-theory:
    corrections to the 3D action → e.g. corrections to the 4D gauge coupling function
  - key properties of F-theory compactifications encoded by **topological terms**
    → might lead to interesting constraints beyond anomaly conditions

- **Many open questions:**
  - M5-branes instantons behave non-trivially in the M-theory to F-theory limit
    [Cvetič,TG,Halverson,Klevers] in progress
  - extend constraint analysis in 4D, including fluxes and potentials
  - constraining continuous parameters
The End.
Thank you!
Gauge theory and singularities

- a closer look at the resolution space: e.g. single stack of branes

4D: - gauge theory on surface 4-cycle $S \subset B_3$
  - further enhancement along intersection curve $\Sigma$ (2-cycle) of two 7-branes
  $\Rightarrow$ matter representations $R$