Fluid dynamics of flapping wings associated with change of domain topology

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in collaboration with

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Topological Fluid Dynamics (IUTAM Symposium)
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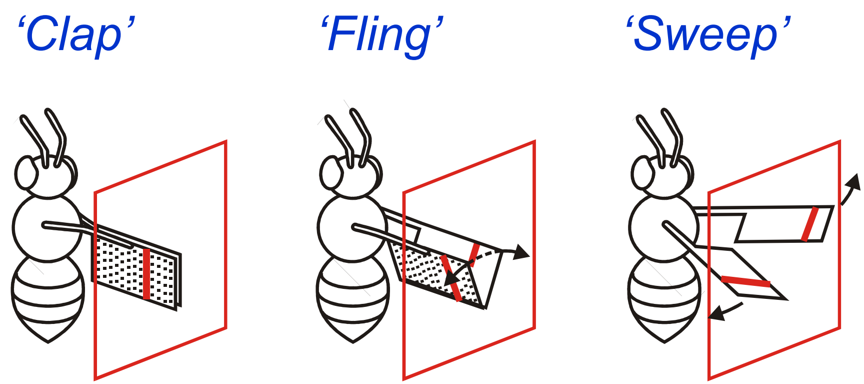
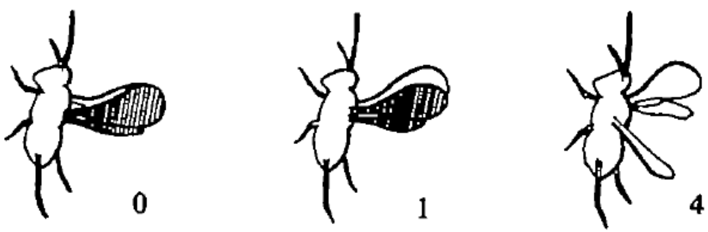
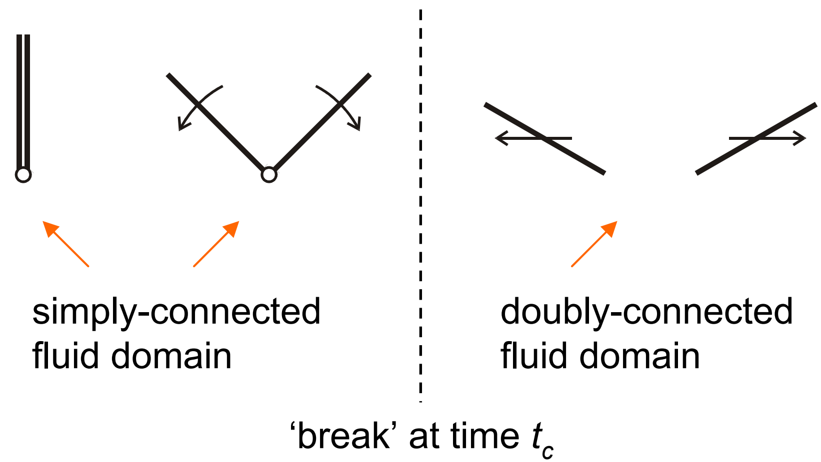
Clap-fling-sweep of *Encarsia formosa*



T. Weis-Fogh, 1973. *J. Exp. Biol.*, **59**



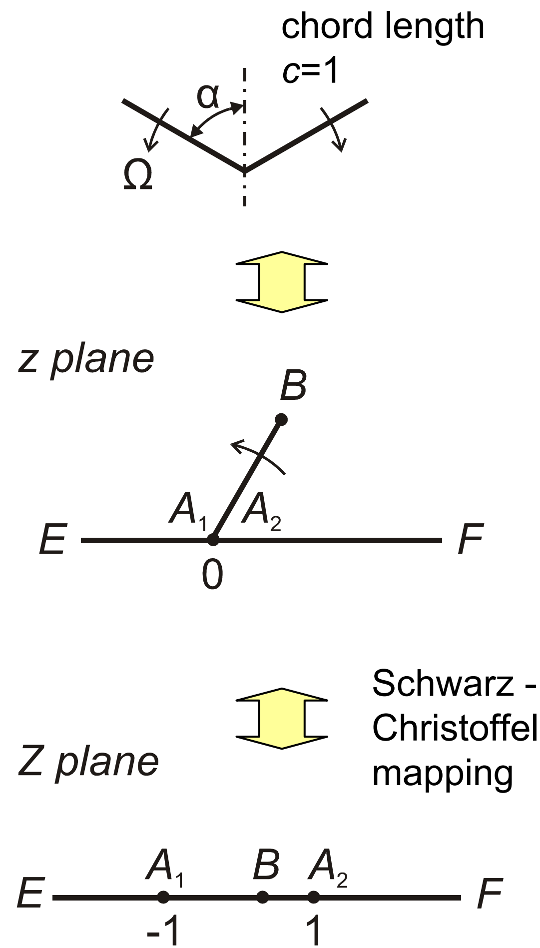
2D approximation



Lighthill's analysis

M. J. Lighthill, 1973. *J. Fluid Mech.*, **60**

- 2D approximation
- Inviscid fluid
- Time before the 'break'



Schwarz-Christoffel mapping

$$z = K\left(1+Z\right)^{1-\alpha/\pi}\left(Z-1\right)^{\alpha/\pi}$$

where $K = K(\alpha, c)$

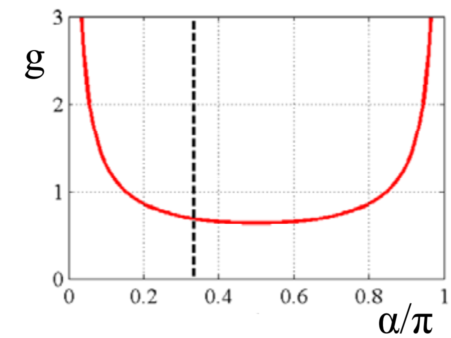
Complex potential

$$w = \frac{\Omega K^2}{2\sin 2\alpha}\Big(-(Z+1)^{2-2\alpha/\pi}(Z-1)^{2\alpha/\pi} + Z^2 + 2Z(1-2\alpha/\pi) + 2(1-2\alpha/\pi)^2 - 1\Big)$$

Potential flow before the break

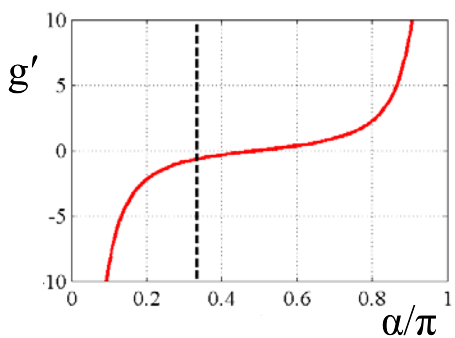
Circulation

$$\Gamma = -\Omega c^2 g(\alpha / \pi)$$



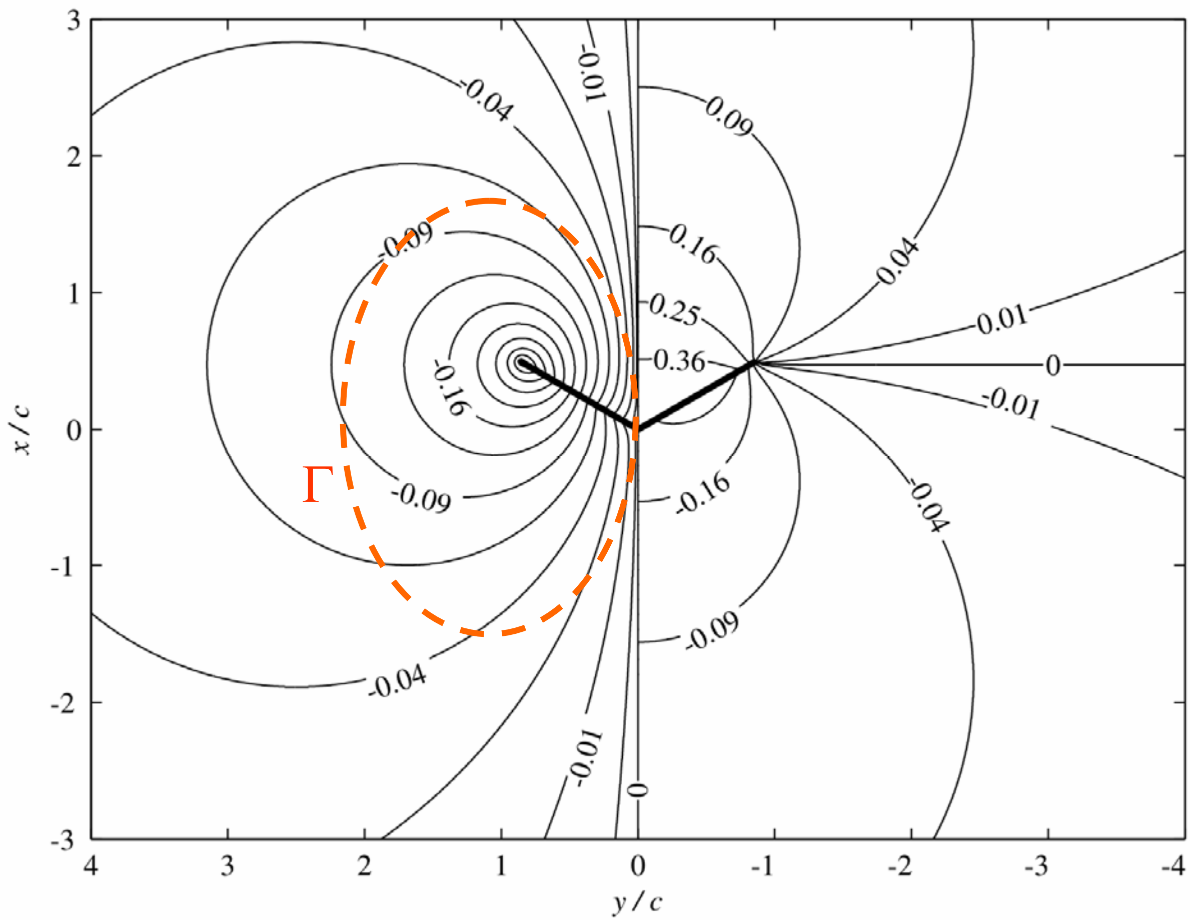
Pressure jump across the hinge

$$\delta p = -\frac{\rho \Omega^2 c^2}{\pi} g'(\alpha / \pi)$$



Streamlines

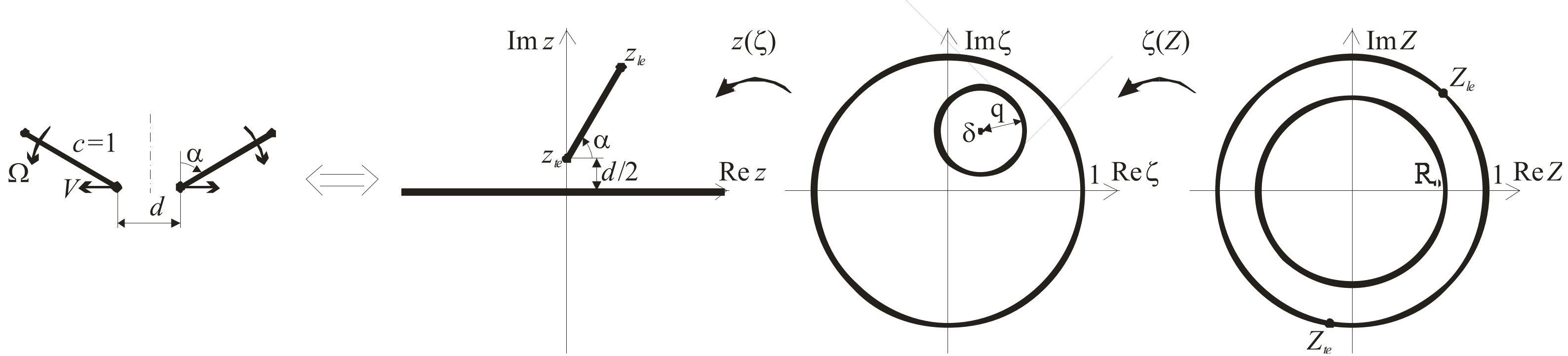
Iso-potential lines



Is the circulation conserved after the break?

Potential flow after the break

Conformal mapping between doubly-connected domains



$$z(\zeta) = \mathrm{i}C_1 \frac{\omega(\zeta,1)}{\omega(\zeta,-1)} + C_2 \qquad \zeta(Z) = \frac{\delta}{|\delta|}\frac{Z - bR_0}{bZ - R_0}$$

R_0, δ, q, C_1, C_2 depend only on α and d

The Schottky – Klein prime function ω is evaluated using a truncated series expansion

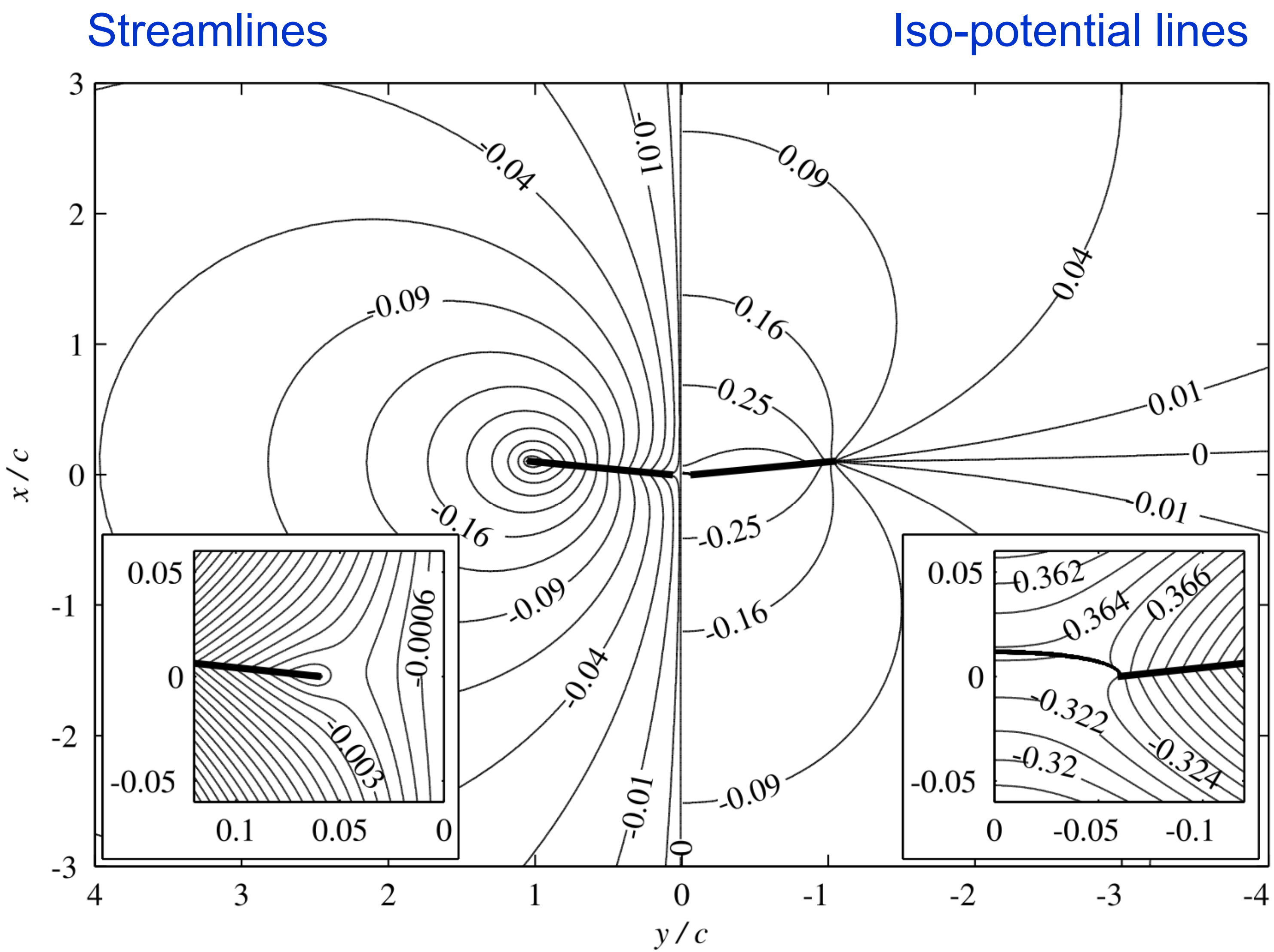
[D.G. Crowdy and J.S. Marshall, 2007. *Comput. Methods Funct. Theory.*, **7**(1)]

Complex potential

$$w = \frac{\Gamma}{2\pi\mathrm{i}}\ln\frac{R_0}{Z} - \frac{1}{2\pi\mathrm{i}}\int_0^{2\pi}(\psi_{te0} - \Omega l_{te}^2/2 - Vl_{te}\cos\alpha)\left(1 - 2\frac{Z}{\mathrm{e}^{\mathrm{i}\theta}}\frac{\omega_\zeta(Z/\mathrm{e}^{\mathrm{i}\theta},1)}{\omega(Z/\mathrm{e}^{\mathrm{i}\theta},1)}\right)\mathrm{d}\theta + C$$

$$l_{te} = |z(\mathrm{e}^{\mathrm{i}\theta}) - z_{te}| \qquad \psi_{te0} = \frac{1}{2\pi}\int_0^{2\pi}(\Omega l_{te}^2/2 + Vl_{te}\cos\alpha)\mathrm{d}\theta$$

Potential flow after the break



Circulation after the break

Circulation

$$\left.\begin{array}{ll}\Gamma = \Gamma_L(t), & t < t_c \\ \Gamma = const, & t > t_c\end{array}\right\}$$

Why would Γ be continuous at t_c?

Flux through the gap between the trailing edges

$$2\psi_{te} = 2\psi_{te0}(\alpha, d, \Omega, V) - \frac{\ln R_0(\alpha, d)}{\pi}\Gamma$$

Suppose $\psi_{te} = 0$

$$\Gamma = \frac{2\pi\psi_{te0}}{\ln R_0} \to \Gamma_L(t_c) \quad \text{as } d \to 0, \quad \text{if } \alpha,\ \Omega,\ V \text{ continuous}$$

Same result if one assumes no singularity at the trailing edge

However, the pressure cannot be continuous in time

$$p = -\rho\left(\frac{\partial\phi}{\partial t} + \frac{|u - \mathrm{i}v|^2}{2}\right), \qquad \frac{\partial\phi}{\partial t} \text{ is related to } \frac{d\Gamma}{dt}$$

Viscous flow near the hinge

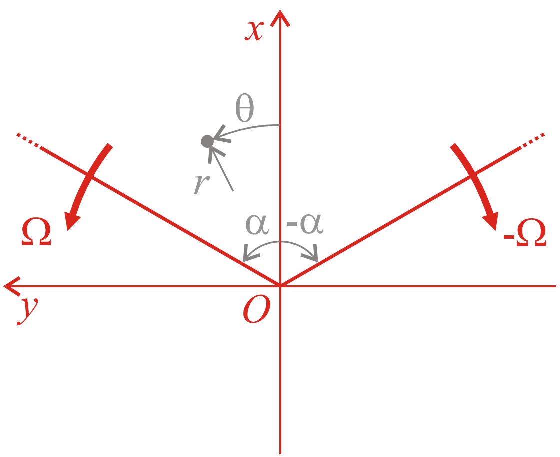
Stokes regime

$$\Delta^2 \psi = 0$$

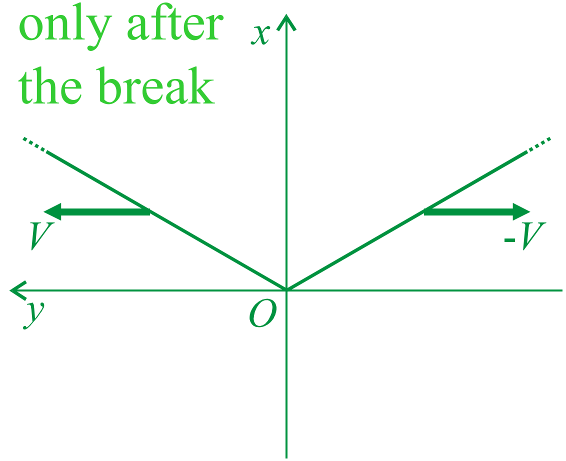
$$\Omega r^2 / \nu \ll 1,\; Vr/\nu \ll 1,$$
$$r \ll c,\; d \ll r$$

$$\psi = \psi_{fling} + \psi_{sweep} + \psi_{Jeffery-Hamel}$$

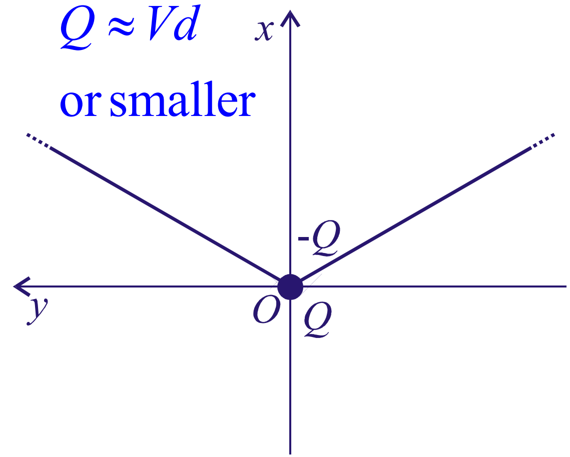
Boundary conditions



non-zero only after the break



negligible, because $Q \approx Vd$ or smaller



Similarity solutions

$$\psi_{fling} = r^2 f_2(\theta) \qquad \psi_{sweep} = r\, f_1(\theta) \qquad \psi_{Jeffery-Hamel} = f_0(\theta)$$

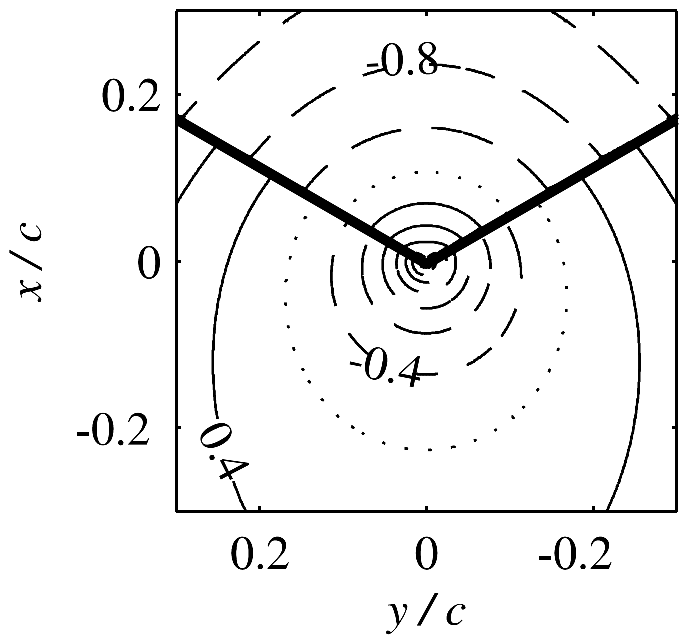
Flow before the break

Local similarity solution

$$\theta \in]0,\alpha[: \quad \psi = -\frac{\Omega r^2}{2}\frac{\sin 2\theta - 2\theta\cos 2\alpha}{\sin 2\alpha - 2\alpha\cos 2\alpha} \qquad \omega = \frac{-4\Omega\theta}{\tan 2\alpha - 2\alpha} \qquad p = \frac{-4\mu\Omega}{\tan 2\alpha - 2\alpha}\ln\frac{C_i}{r}$$

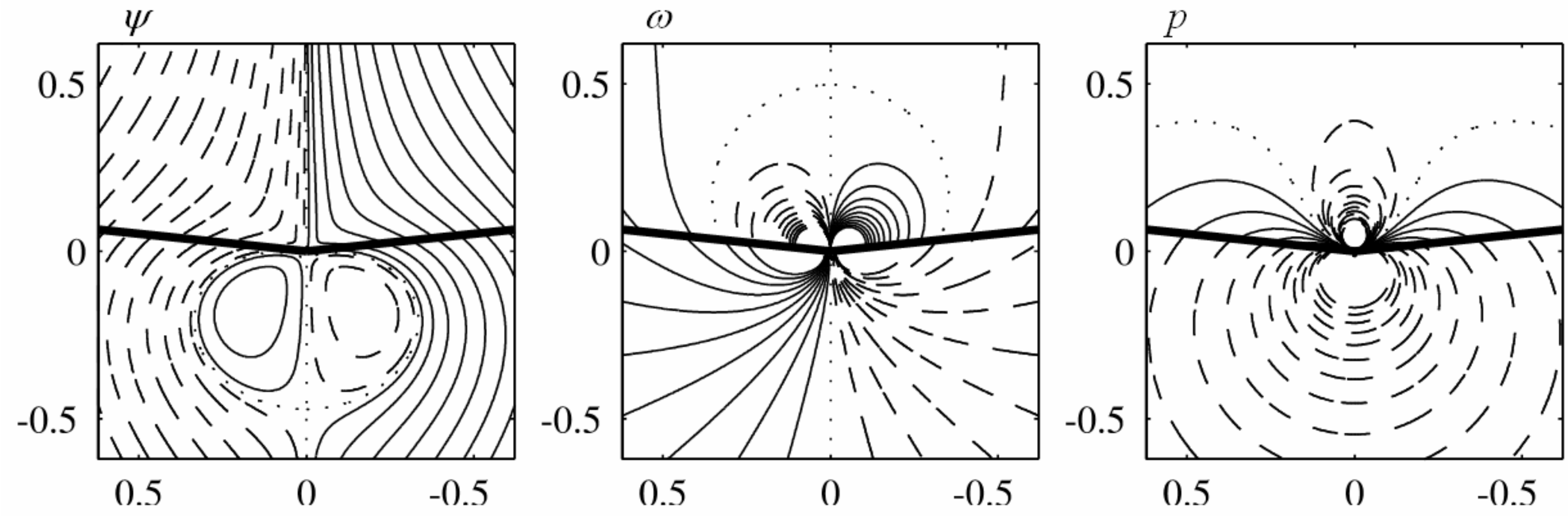
$$\theta \in]\alpha,\pi[: \quad \psi = -\frac{\Omega r^2}{2}\frac{\sin 2\theta + 2(\pi-\theta)\cos 2\alpha}{\sin 2\alpha + 2(\pi-\alpha)\cos 2\alpha} \qquad \omega = \frac{4\Omega(\pi-\theta)}{\tan 2\alpha + 2(\pi-\alpha)} \qquad p = \frac{-4\mu\Omega}{\tan 2\alpha + 2(\pi-\alpha)}\ln\frac{C_e}{r}$$

Numerical simulation at *Re* = 1
Pressure iso-contours:

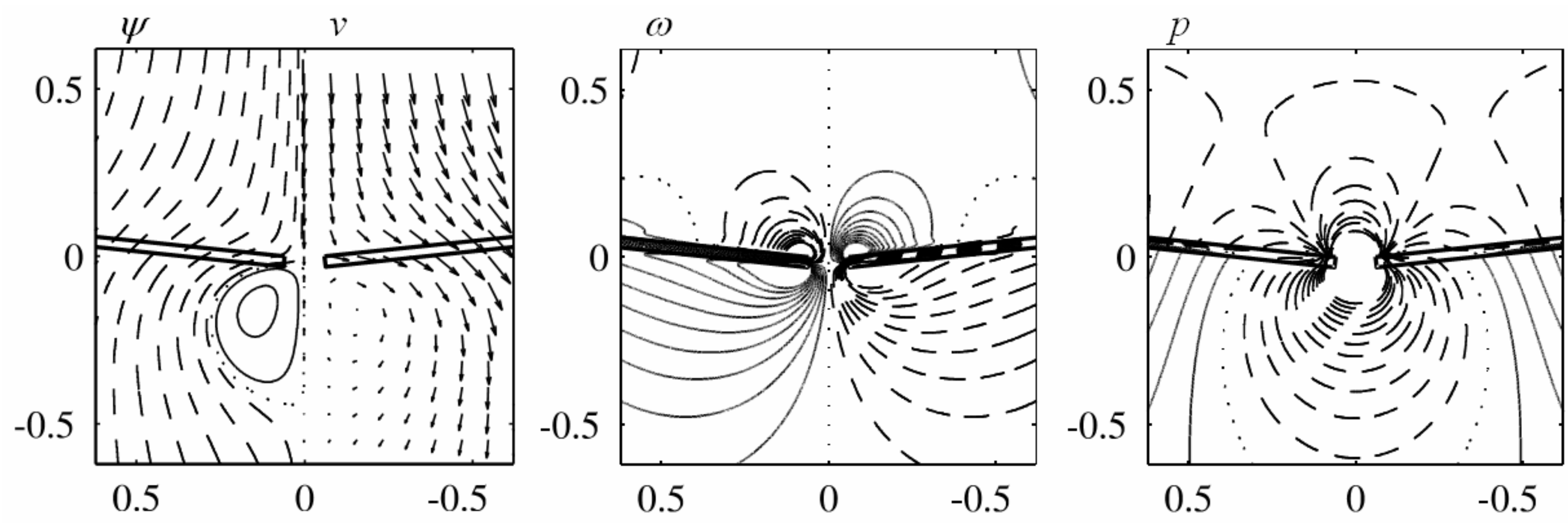


Flow after the break

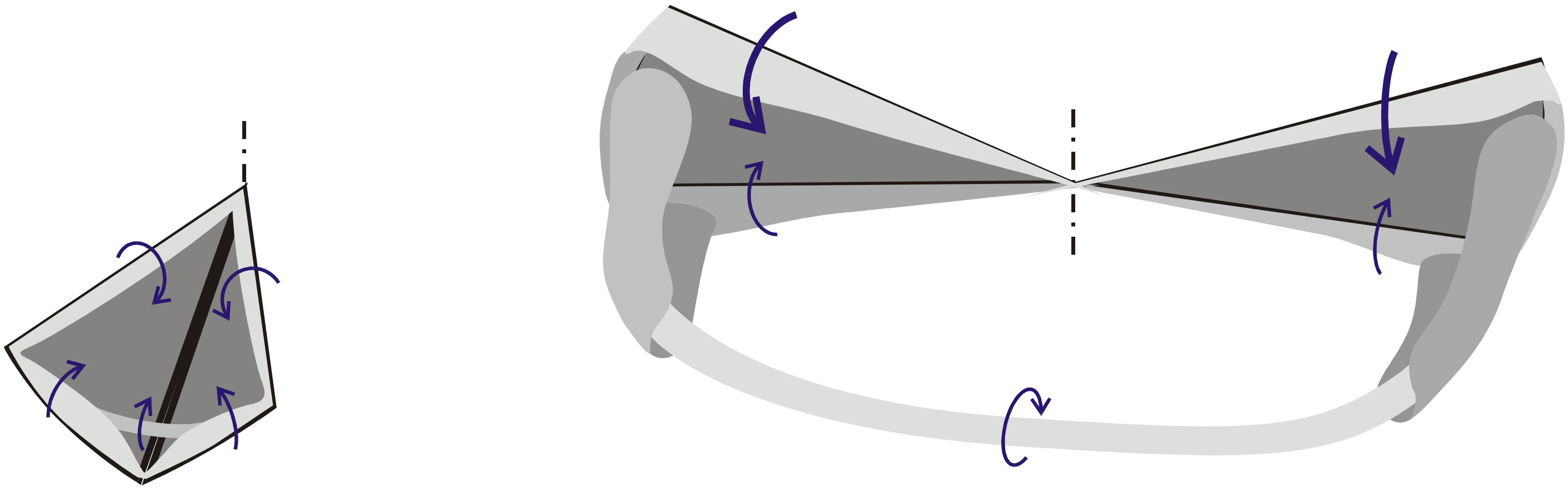
Local solution



Numerical simulation at *Re* = 1



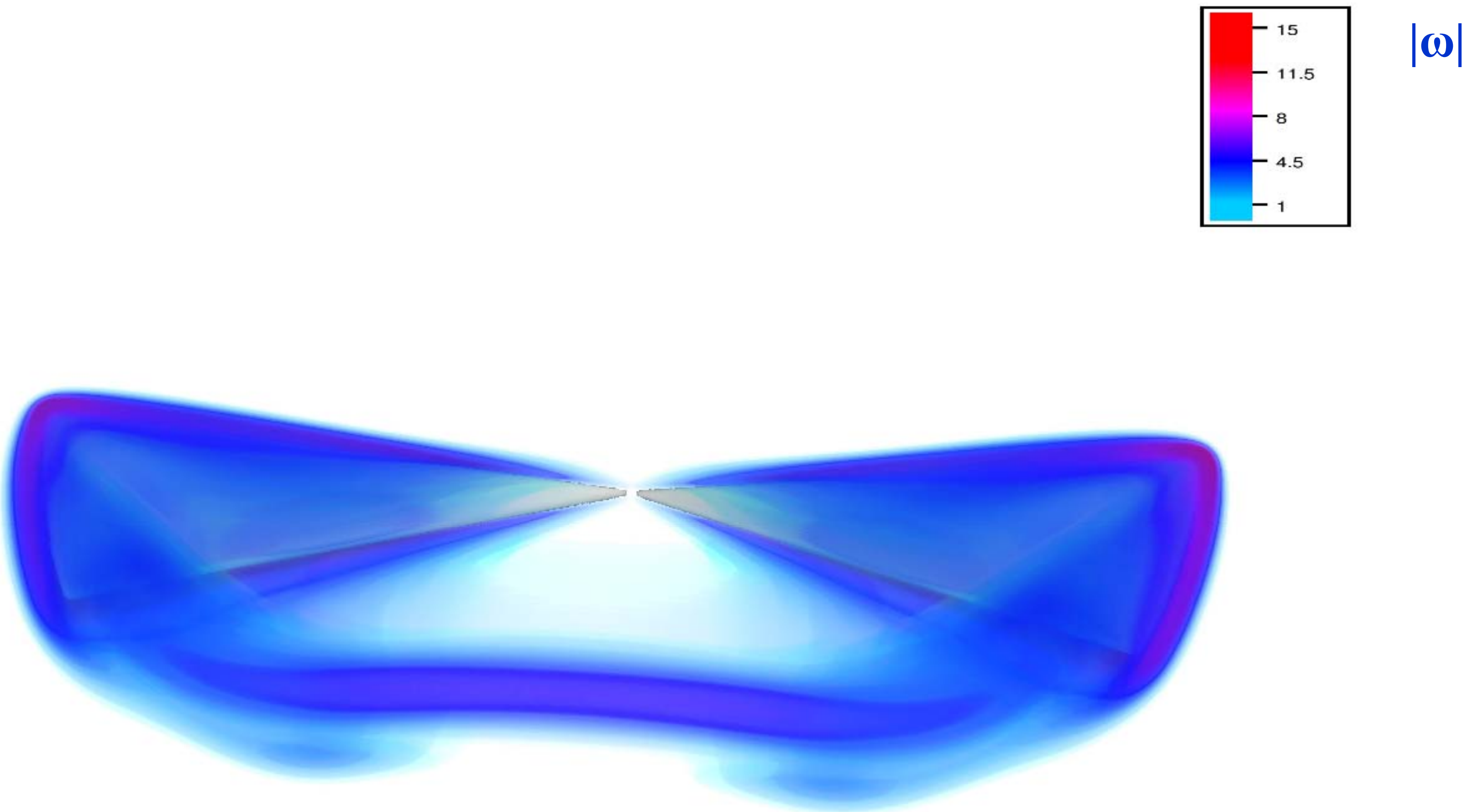
Three-dimensional flow



Conclusions

- In an inviscid fluid, the complex potential undergoes a transition from single-valued to multi-valued.
- One may require a continuous evolution of the circulation in time. However, the potential flow theory does not provide a physical justification.
- In a viscous fluid, the flow near the trailing edges is controlled by the viscous stresses that inhibit the flux through the opening gap.
- In a three-dimensional flow, a non-zero circulation about the wings is accompanied by a wing-tip vortex.

Three-dimensional flow at Re = 128



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