Asymptotic higher ergodic invariants of magnetic lines

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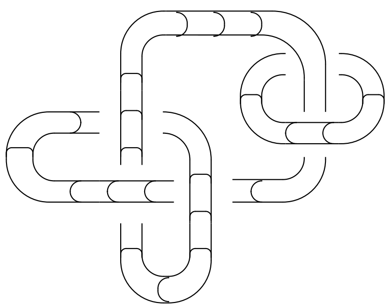
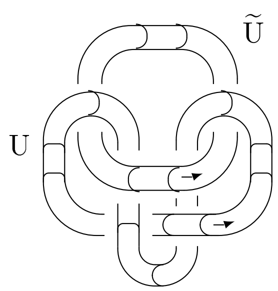
Magnetic tubes





Milnor's Invariants for magnetic tubes are applied

Mikhail I. Monastyrsky and Vladimir S. Retakh, „*Topology of linked defects in condensed matter*“, Comm. Math. Phys. 103, no. 3.

An example of the Milnor's Invariant: the Sato-Levine integral

jointly with A. Ruzmaikin „*A forth-order topological invariant of magnetic or vortex lines*“, Journal of Geometry and Physics, vol.15 (1994).

This integral is well-defined for a pair of unlinked magnetic tubes.

$$\mathbf{A}_i, \quad \mathrm{rot}\mathbf{A}_i = \mathbf{B}_i, \quad i = 1, 2.$$

$$\mathbf{F} = \mathbf{A}_1 \times \mathbf{A}_2 + \mathbf{grad}\phi_2\mathbf{B}_1 - \mathbf{grad}\phi_1\mathbf{B}_2.$$

$$\mathrm{div}(\mathbf{F}) = 0, \quad \exists\mathbf{G}, \quad \mathrm{rot}\mathbf{G} = \mathbf{F}.$$

$$W = \int_{\mathbb{R}^3} (\mathbf{F}, \mathbf{G})dx + \int \varphi_2^2(\mathbf{A}_1, \mathbf{B}_1) + \int \varphi_1^2(\mathbf{A}_2, \mathbf{B}_2).$$

The Sato-Levine integral is invariant in the ideal MHD.

KH Moffatt (2001)

The invariant of Sato-Levine
Is something that ought to be seen;
I perform some high jinks:
With the knots and the links;
I hope that you see what I mean!

An useful modification of the Sato-Levine invariant of 3 magnetic tubes

by the author *„On a new integral formula for an invariant of 3-component oriented links“*, Journal of Geometry and Physics, 53 (2005). This is a Milnor's type invariant, order 12 in the sense of V.A. Vassiliev, is well-defined for an arbitrary configuration of 3 magnetic tubes.

$$\mathbf{A}_i, \quad \mathrm{rot}\mathbf{A}_i = \mathbf{B}_i, \quad i = 1, 2, 3.$$

$$\mathbf{F} = (1,3)(2,3)\mathbf{A}_1 \times \mathbf{A}_2 + (2,1)(3,1)\mathbf{A}_2 \times \mathbf{A}_3 + (3,2)(2,1)\mathbf{A}_3 \times \mathbf{A}_1$$

$$-\mathbf{grad}\phi_1\mathbf{B}_1 - \mathbf{grad}\phi_2\mathbf{B}_2 - \mathbf{grad}\phi_3\mathbf{B}_3.$$

$$\mathrm{div}(\mathbf{F}) = 0, \quad \exists\mathbf{G}, \quad \mathrm{rot}\mathbf{G} = \mathbf{F}.$$

$$M = \int_{\mathbb{R}^3} (\mathbf{F}, \mathbf{G})dx + \dots,$$

where $\dots$ means terms, as in the formula of W.

A problem by V.I.Arnol'd

V.I.Arnol'd in 1984 formulated the following problem: "To transform asymptotic ergodic definition of Hopf invariant of a divergence-free vector field to Novikov's theory, which generalizes Withehead product in homotopy groups“.

Asymptotic invariants of magnetic fields, in particular, the theorem by V.I.Arnol'd about asymptotic Gaussian linking number, is a bridge, which relates differential equitations and topology.

Asymptotic ergodic Hopf invariant by V.I.Arnol'd (1974)

Assume B is magnetic field inside a finite volume in $\mathbb{R}^3$. Define the Gaussian asymptotic linking number of the magnetic lines of $\mathbf{B}$ by the times T_1, T_2 correspondingly, issued from the points x_1, x_2 correspondingly, by the following formula:

$$\Lambda_{\mathbf{B}}(T_1, T_2; x_1, x_2) = \frac{1}{4\pi T_1 T_2} \int_0^{T_2} \int_0^{T_1} \frac{\langle \int \dot{x}_1(t_1), \dot{x}_2(t_2), x_1(t_1) - x_2(t_2) \rangle}{\| x_1(t_1) - x_2(t_2) \|^3} dt_1 dt_2,$$

$$\Lambda_{\mathbf{B}}(x_1, x_2) = \lim_{T_1, T_2 \to +\infty} \Lambda_{\mathbf{B}}(T_1, T_2; x_1, x_2),$$

where $x_i(t_i) = g^t(x_i)$ is the magnetic line, issued from the point x_i, $i = 1, 2$, and $\dot{x}_i(t_i) = \frac{d}{dt_i} g^{t_i} x_i$ are the corresponded velocity tangent vectors ($=$ the vectors of the magnetic field).

The limit $\Lambda_{\mathbf{B}}(x_1, x_2)$ exists almost everywhere on $D \times D \subset \mathbb{R}^3 \times \mathbb{R}^3$. The function $\Lambda_{\mathbf{B}}(x_1, x_2)$ belongs L^1 (is integrable) and the following equation:

$$\int \Lambda_{\mathbf{B}}(x_1, x_2) dx_1 dx_2 = \chi_{\mathbf{B}}$$

is well-defined, where the right side of the equation is given by the formula:

$$\chi_{\mathbf{B}} = \int (\mathbf{A}, \mathbf{B}) dD, \quad \mathrm{rot} \mathbf{A} = \mathbf{B}.$$

The value is called the asymptotic Hopf invariant, or, the helicity integral. This invariant has the dimension $G^2 sm^4$, and the corresponding combinatorial invariant of links, the linking number, has the topological order 1. The function $\Lambda(x_1, x_2)$ is called the helicity density.

Definition of asymptotic ergodic invariants of magnetic fields

Let Ω be the space of all magnetic fields, which are structured into magnetic tubes. Assume $I : \Omega \to \mathbb{R}$ is a functional.

The configuration space K is defined as following. Assume that a collection of r magnetic lines $L_1, \dots, L_r$ of the magnetic field $\mathbf{B}$, which is parametrized of the segments $[0, T]$, started at the prescribed points $\{l_1, \dots, l_r\}$ of the domain Ω correspondingly. A subcollection $\{l_1; x_1, \dots, x_q\}$ of the collection Ψ consists of q points, which are on the first magnetic line L_1 of the magnetic field $\mathbf{B}$, the following collection $\{l_2; x_{q+1}, \dots, x_{2q}\}$ of the collection Ψ consists of q points, each point belongs to the second magnetic line L_2 of $\mathbf{B}$, e.t.c., the last subcollection $\{l_r; x_{q(r-1)+1}, \dots, x_{qr}\}$ of the collection Ψ consists of q points, each point belongs to the r-th magnetic line L_r of $\mathbf{B}$. Each point x_{qj+i} is well-defined by the time-variable t_{qj+i}, $1 \le j \le r$, $1 \le i \le q-1$, $0 \le t_{qj+i} \le T$, which is the time of the evolution of the point l_j into the point x_{qj+i} by the magnetic flow. Let us call I is asymptotic ergodic finite-order functional

The formal functional I of a finite-order is called asymptotic, if this functional is defined as the limit $T \to +\infty$ of integrals (called Cesaro averages) over all finite collections $\Psi = \{l_1; x_1, \dots, x_{qr}\}$ of $qr + 3$ points in a configuration space K.

Asymptotic ergodic invariant of magnetic field is a functional, which is defined using an integral formula of a finite type invariant of links.

Remark

1. An asymptotic ergodic invariant is defined as the ergodic mean value of an integrable function on a configuration apace K, equipped with r commuted flows.

2. A finite-type invariant M is defined using a Massey-type integral. In the asymptotic limit there is no problem with a particular- and multi- values of Massey-type integrals.

Return Condition

Assume $\mathbf{B}$, $\mathrm{div}(\mathbf{B}) = 0$, is a magnetic filled in a magnetic tube (the field is tangent to the surface of the tube). Let $\{g^t : S \to S\}$ be the magnetic flow. Let us call $\mathbf{B}$ satisfies the return condition if there exists $t_0 > 0$, $t_0 = t_0(x)$, such that

$$g^{t_0}(x) = x.$$

Main Theorem

There exists an asymptotic ergodic invariant M for generic magnetic fields, which admits a combinatorial meaning for magnetic fields, satisfied Return Condition.

Remark about a proof of Main Theorem

We apply an integral M for each triple of magnetic lines (not tubes!) and takes the asymptotic limit, when the length of each line tends to $+\infty$. The asymptotic functional M contains elementary terms of the following 3 types:

1. $(1,2)^2$ the square of the linking number of two magnetic lines.
2. The Gauss integral of the pair of fields $(\mathbf{A}_1 \times \mathbf{A}_2; \mathbf{A}_1 \times \mathbf{A}_2)$, where $\mathbf{A}_i$, $i = 1, 2$ are the vector-potentials of the magnetic line L_i.
3. Integrals $\int \varphi^2(\mathbf{A}_1, \mathbf{B}_1) dL_1$, where $\mathbf{grad}(\varphi) = (1,2)\mathbf{A}_2 - (2,3)\mathbf{A}_3$, φ is the common scalar potential of the magnetic lines L_2, L_1 over the magnetic line L_1. A scalar potential of such a type was investigated by M.Berger.

Remark

Gauge transformations of terms of the type 2 and 3 in asymptotic limits are not investigated (could be complicated).

Example

Assume that a magnetic line $L(y)$ of $\mathbf{B}(x)$ is dense in a magnetic tube U. At each point $y \in L$ consider the vector-potential

$$\mathbf{A}(x;y) = \frac{\mathbf{B}(y) \times (x-y)}{4\pi||x-y||^3}$$

of the corresponding elementary magnetic dipole

$$\mathrm{rot}_x(\mathbf{A}(x;y)).$$

At each point $x \in \mathbb{R}^3$ the mean value of the y-family $\mathbf{A}(x;y)$

$$m_y[\mathbf{A}(x;y)] = \int \mathbf{A}(x;y)dy = \mathbf{A}(x)$$

is well-defined.

At each point $x \in \mathbb{R}^3$ the principal mean value of the y-family $\mathrm{rot}_x(\mathbf{A}(x;y))$

$$m_y[\mathrm{rot}_x(\mathbf{A}(x;y))] = \int \mathrm{rot}_x(\mathbf{A}(x;y))dy = \tilde{\mathbf{B}}(x),$$

is well-defined.

The operators rot, m_y are not commuted! We get:

$$\mathrm{rot}\mathbf{A}(x) = \mathbf{B}(x),$$

$$\tilde{\mathbf{B}}(x) = \frac{1}{3}\mathbf{B}(x).$$

q–monomial helicities

Define the asymptotic q-monomial linking coefficient $\Lambda^{[q]}_{\mathbf{B}}$ and the asymptotic ergodic invariant $\chi^{[q]}_{\mathbf{B}}$, which is called q-helicity. Define q-linking coefficient $\Lambda^{[q]}_{\mathbf{B}}(T_1,T_2;x_1,x_2)$ of the magnetic lines of $\mathbf{B}$ by the times T_1,T_2, which are issued from the points x_1,x_2, as follows:

$$\Lambda^{[q]}_{\mathbf{B}}(T_1,T_2;x_1,x_2) = \tag{1}$$

$$\frac{1}{4^q\pi^qT_1^qT_2^q}\int_0^{T_1}\dots\int_0^{T_1}\int_0^{T_2}\dots\int_0^{T_2}\frac{\langle \dot{x}_{1,1}(t_{1,1}),\dot{x}_{2,1}(t_{2,1}),x_{1,1}(t_{1,1})-x_{2,1}(t_{2,1})\rangle}{\|x_{1,1}(t_{1,1})-x_{2,1}(t_{2,1})\|^3}\cdot\dots\cdot$$
$$\frac{\langle \dot{x}_{q,1}(t_{1,q}),\dot{x}_{q,2}(t_{2,q}),x_{1,q}(t_{1,q})-x_{2,q}(t_{2,q})\rangle}{\|x_{1,q}(t_{1,q})-x_{2,q}(t_{2,q})\|^3}dt_{1,1}\dots dt_{1,q}dt_{2,1}\dots dt_{2,q}.$$

In this integral we may put $T_1=T_2=T$, this gives a little simplification.

For almost arbitrary pair of points x_1,x_2 there exists a limit

$$\Lambda^{[q]}_{\mathbf{B}}(x_1,x_2) = \lim_{T\to+\infty}\Lambda^{[q]}_{\mathbf{B}}(T;x_1,x_2), \tag{2}$$

where the function in the right side of the equation is integrable on $U\times U$.

The formula (2) determines the time-average of the corresponding integral kernel, which is denoted by $\Gamma^{[q]}$. In the particular case $q=1$ the formula (1) coincides with the formula by V.I.Arnol'd for the asymptotic linking number of magnetic lines.

Let us define (a formal) functional, which generates the q–helicity by the formula:

$$\chi^{[q]}_{\mathbf{B}} = \int\Lambda^{[q]}_{\mathbf{B}}(x_1,x_2)dx_1dx_2,$$

where $\Lambda^{[q]}_{\mathbf{B}}(x_1,x_2)$ is defined in (2).

Theorem

Assume $\mathbf{B}$ is a magnetic field in a magnetic tube U.

–1. The q-monomial linking number $\Lambda^{[q]}_{\mathbf{B}}(x_1,x_2)$ is well-defined almost everywhere, except, possibly, a subset in $U \times U$ of zero measure.

–2. The function $\Lambda^{[q]}_{\mathbf{B}}(x_1,x_2) = (\Lambda_{\mathbf{B}}(x_1,x_2))^q$ is integrable and the integral

$$\chi^{[q]}_{\mathbf{B}} = \int \Lambda^{[q]}_{\mathbf{B}}(x_1,x_2)dx_1dx_2$$

is an invariant in the ideal MHD.

Proof of Theorem

Let us define the configuration space $K_{2,q} = U\times\mathbb{R}^k\times U\times\mathbb{R}^k$ and the mapping, which is called the evolution mapping:

$$F_q : K_{2,q} \to (U)^q \times (U)^q, \tag{3}$$

where (U^q) is the Cartesian product of q exemplars of a magnetic tube U (or, a finite number of magnetic tubes), by the formula:

$$F_q(x_1,t_{1,1}\dots t_{1,q},x_2,t_{2,1}\dots t_{2,q}) =$$

$$(g^{t_{1,1}}(x_1),\dots g^{t_{1,q}}(x_1),g^{t_{2,1}}(x_2)),\dots g^{t_{2,q}}(x_2))$$

where g^t is the magnetic flow. Therefore, the configuration space $K_{2,q}$ is defined as the space of two ordered collections of q points, each collection on the corresponded magnetic line.

On the space $K_{2,q}$ a collection of $2q$ flows (each two flows are commuted) along the coordinates $\mathbb{R}_{1,i},\mathbb{R}_{2,j}$, $1 \le i \le q$, $1 \le j \le q$. This coordinates are called time-coordinates and are denoted by $t_{1,i}$, $t_{2,j}$, the images $g^{t_{1,i}}(x_1)$, $g^{t_{2,j}}(x_2)$ are denoted by $x_{1,i}(t_{1,i})$, $x_{2,j}(t_{2,j})$, or, briefly, by $x_{1,i}$, $x_{2,j}$. The standard volume form on $K_{2,q}$ denote by $dK_{2,q}$. Let us denote by

$$\Gamma^{[q]} : K_{2,q} \to \mathbb{R}. \tag{4}$$

the integral kernel in (1). The kernel $\Gamma^{[q]}$ determines a functional.

Prove Statement 1

Let us apply for each of $2q$ flows on the configuration space $K_{2,q}$ the the Birkhoff Theorem. Let us prove that the average value of the function $\Gamma^{[q]}$ along almost each $2q$-dimensional parametrization of a pair of magnetic lines has the limit, for almost everywhere. Statement 1 is proved.

Prove Statement 2

Denote by $K_{2,q;T} \subset K_{2,q}$ a compact subspace in the configuration space, for which each time-variable coordinate belongs to the segment $[0,T]$. Let us formulate the definition of limiting tensor.

Definition

Assume a function

$$A^{[q]} : K_{2,q} \to \mathbb{R} \tag{5}$$

is integrable on each subspace $K_{2,q;T} \subset K_{2,q}$. Let us say that integrable non-negative function

$$a^{[q]} : U \times U \to \mathbb{R}_+, \tag{6}$$

which tends to $+\infty$, when a point to the origin tends to the diagonal $U \times U \supset \{(x_1,x_2)|_{x_1=x_2}\}$, is called a limiting tensor for (5), if there exists $T_0 \geq 0$, such that for an arbitrary $T > T_0$ the functions $a^{[q]} \circ p : K(2,q) \to (U \times U) \to \mathbb{R}_+$ (this function is integrable, because the function (6) is integrable) and $|A^{[q]}| : K_{2,q} \to \mathbb{R}_+$ are satisfied the following equation:

$$\int |A^{[q]}| dK_{2,q;T} \leq \int a^{[q]} \circ p \quad dK_{2,q;T}. \tag{7}$$

Lemma

Assume that for an arbitrary point $(x_1,x_2) \in K_{2,q}$ there exists a mean value over time-variables of the function (5). The mean value over time-variables determines an integrable function, which is denoted by $\bar{A}^{[q]}(x_1,x_2) : U \times U \to \mathbb{R}$. Assume that there exists a limiting tensor (6). Then the function $\bar{A}^{[q]}(x_1,x_2)$ is integrable (belongs to the space L^1).

Apply Lemma and construct a limiting tensor $\delta^{[q]}$ for the sub-integral kernel $\Gamma^{[q]}$ (4).

Take a smoothing of the subintegral kernel $\Gamma^{[q]}$, using a small finite parameter $\varepsilon > 0$, over this additional time-parameter the integration of $\Gamma^{[q]}$ is well-defined. Each time-variable vary by additional small alterations in the interval $[-\varepsilon, +\varepsilon]$. Denote the result of the smoothing of $\Gamma^{[q]}$ by $K_\varepsilon(x_{1,1}, x_{2,1}, \dots, x_{1,q}, x_{2,q})$.

Replace the integral kernel $\Gamma^{[q]}$ into the smoothed kernel K_ε in the right side (below the limit of the equation (2). The mean value over the time-variables over the segment $[0, T]$ at the left side of the non-equality (7), which is used for $A^{[q]} = K_\varepsilon$, is the following:

$$\Lambda^{(q)}_{\mathbf{B}}(T_1, T_2; x_1, x_2; \varepsilon) = \tag{8}$$
$$\frac{1}{2^q \pi^q T_1^q T_2^q} \int_0^{T_1} \dots \int_0^{T_1} \int_0^{T_2} \dots \int_0^{T_2}$$
$$K_\varepsilon(x_{1,1}, x_{2,1}, \dots, x_{1,q}, x_{2,q}) dt_{1,1} \dots dt_{1,q} dt_{2,1} \dots dt_{2,q},$$

where the integral kernel K_ε is calculated by the formula:

$$K_\varepsilon(x_{1,1}, x_{2,1}, \dots, x_{1,q}, x_{2,q}) = \tag{9}$$
$$\varepsilon^{-q} \int_0^\varepsilon \dots \int_0^\varepsilon \int_0^\varepsilon \dots \int_0^\varepsilon \frac{\langle \dot{x}_{1,1}(t_{1,1}), \dot{x}_{2,1}(t_{2,1}), x_{1,1}(t_{1,1}) - x_{2,1}(t_{2,1}) \rangle}{|x_{1,1}(t_{1,1}) - x_{2,1}(t_{2,1})|^3} \cdot \dots$$
$$\cdot \frac{\langle \dot{x}_{q,1}(t_{1,q}), \dot{x}_{q,2}(t_{2,q}), x_{1,q}(t_{1,q}) - x_{2,q}(t_{2,q}) \rangle}{|x_{1,q}(t_{1,q}) - x_{2,q}(t_{2,q})|^3} dt_{1,1} \dots dt_{1,q} dt_{2,1} \dots dt_{2,q}.$$

Put is the right side of the non-equality (7) the expression $A^{[q]} = \Gamma^{[q]}$:

$$\int |\Gamma^{[q]}| dK_{2,q;T}. \tag{10}$$

Put in the same formula the expression $A^{[q]} = K_\varepsilon^{[q]}$:

$$\int |K_\varepsilon^{[q]}| dK_{2,q;T}. \tag{11}$$

Obviously, the integrals (10), (11) distinguishes by its absolute value by a real, which is non-depended of T and is arbitrary small, if $\varepsilon \to 0$. The difference of this two integrals are given by boundary conditions, when one of the parameter belongs to $\{0, T\}$. Therefore for the proof of the required statement is sufficient to construct a limiting tensor for $K_\varepsilon^{[q]}$ for an appropriate finite $\varepsilon > 0$.

The smoothed integral kernel (9) is equal to a product of the integrals as following:

$$K_\varepsilon(x_{1,1}, x_{2,1}, \dots, x_{1,q}, x_{2,q}) = \tag{12}$$

$$\varepsilon^{-q} \prod_{j=1}^{q} \int_0^\varepsilon \int_0^\varepsilon \frac{\langle \dot{x}_{1,j}(t_{1,j}), \dot{x}_{2,j}(t_{2,j}), x_{1,j}(t_{1,j}) - x_{2,j}(t_{2,j}) \rangle}{|x_{1,j}(t_{1,j}) - x_{2,j}(t_{2,j})|^3} dt_{1,j} dt_{2,j}.$$

Let us apply an elementary non-equality:

$$K_\varepsilon(x_{1,1}, x_{2,1}, \dots, x_{1,q}, x_{2,q}) \le \tag{13}$$

$$\frac{1}{q\varepsilon^q} \sum_{j=1}^{q} (\int_0^\varepsilon \int_0^\varepsilon \left| \frac{\langle \dot{x}_{1,j}(t_{1,j}), \dot{x}_{2,j}(t_{2,j}), x_{1,j}(t_{1,j}) - x_{2,j}(t_{2,j}) \rangle}{\| x_{1,j}(t_{1,j}) - x_{2,j}(t_{2,j}) \|^3} \right| dt_{1,j} dt_{2,j})^q.$$

Let us fix an arbitrary small constant $1 >> \delta_0 > 0$, such that at the prescribed scale the magnetic field and its partial derivatives are of a small variation. Assume that points $x_{1,j}$, $x_{2,j}$ are distinguished not more then to a positive small constant δ_0, then the corresponding term in (13) is absolutely estimated by the following term

$$C \ln^q(\rho_{\mathbf{B}}(x_{1,j}, x_{2,j})), \tag{14}$$

where $\rho_{\mathbf{B}}(x_{1,j}, x_{2,j})$ is the distance from the point $x_{1,j}$ to the magnetic line, which contains the point $x_{2,j}$. The coefficient $C < 0$, in the case $q = 2s + 1$, and $C > 0$, in the case $q = 2s$, depends of first and second order partial derivatives of $\mathbf{B}$ and of the prescribed constant δ_0.

If the distance between points $x_{1,j}$, $x_{2,j}$ is greater then δ_0, the corresponding term in the expression (13) is absolutely estimated by a positive constant, which is not depended of δ_0 and of the components of $\mathbf{B}$. Putting the non-equalities (13), (14) into the expression (12). This gives the following absolute bound of the integral kernel (9) of the integral (8):

$$K_\varepsilon(x_{1,1}, x_{2,1}, \dots, x_{1,q}, x_{2,q}) \le \frac{C^q}{\varepsilon^q} \sum_{i=1}^{q} \ln^q(\rho_{\mathbf{B}}(x_{1,i}, x_{2,i})). \tag{15}$$

Define the limiting tensor by the formula:

$$\delta^{[q]}(x_1, x_2) = \frac{qC^q}{\varepsilon^q} \ln^q(\rho_{\mathbf{B}}(x_1, x_2)).$$

Because the integral

$$\int \ln^q(\rho_{\mathbf{B}}(x_1, x_2))dx_1 dx_2$$

over an arbitrary compact domain in $\mathbb{R}^3(x_1) \times \mathbb{R}^3(x_2)$ exists, for an arbitrary $q \geq 1$ the function in the left side of (15) is integrable. The required estimation (7) for $A^{[q]} = K_{\varepsilon}^{[q]}$ follows from the equation (15): using this equation each term in the expression (13) is estimated. This proves Statement 2 and the Theorem.

Conclusion remarks

1. The 2-monomial helicity $\chi^{(2)}$ is the dispersion on the magnetic helicity density.

2. The q-monomial helicity $\chi^{(q)}$ is a higher momentum of the magnetic helicity density.

3. The 2-monomial helicity $\chi^{(2)}$ continuously, even with a Lipschitz coefficient (but not smoothly!) depends of $\mathbf{B}$. Lipschitz coefficients in particular cases are calculated. This gives an application for the induction equation with the α-term and with the diffusion term.

4. The configuration space for the asymptotic ergodic invariant M is very high-dimensional.

5. If two magnetic lines L_1, L_2 from the triple L_1, L_2, L_3 are ergodic in a common magnetic tube, then $M(L_1, L_2, L_3)$ is trivial (Conjecture).