\( \mathbb{CP}^2 \) (baby) Skyrmions in three component superconductors

Originially: Topological solitons in multi-component superconductors: from baby-Skyrmions to vortex loops.

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with

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Quantised Flux in Tightly Knotted and Linked Systems – Dec, 7-th, 2012
based on J. Garaud, J. Carlström, E. Babaev and M. Speight

Topological solitons in three-band superconductors with broken time reversal symmetry,

Chiral $\mathbb{CP}^2$ skyrmions in three-band superconductors,

Length scales, collective modes, and type-1.5 regimes in three-band superconductors,

Additional (movie) material
http://people.umass.edu/garaud/3CGL-soliton.html
Motivations

Growing importance of multi-component/band superconductors

- Growing number of known multi-component-multi-band material
- Iron pnictides are possibly described by more than two superconducting components.
- Possible states with time reversal symmetry breakdown.
  - T. K. Ng, N. Nagaosa; Europhys. Lett. 87, 17 003 (2009)
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Can be achieved with long Josephson junctions (or bilayers)

- by investigating Josephson effect between a two-band (with $s_{\pm}$ pairing) and a conventional $s$-wave superconductors.
- the $s_{\pm}$ interband couplings, lock phases to $\pi$
  $\Rightarrow$ frustration induced Broken TRS state (BTRS)
Multicomponent Ginzburg-Landau model

Three complex scalars \( \psi_a = |\psi_a| \exp i\varphi_a \) are the SC condensate

\[
F_{3\text{GL}} = \frac{1}{2} (\nabla \times A)^2 + \frac{1}{2} \sum_{a=1}^{3} |(\nabla + i e A)\psi_a|^2 + (2\alpha_a + \beta_a |\psi_a|^2)|\psi_a|^2
- \sum_{b>a}^{3} \eta_{ab} |\psi_a| |\psi_b| \cos \varphi_{ab}
\]

where \( \varphi_{ab} \equiv \varphi_b - \varphi_a \).
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- 3 self-interacting complex fields, $\psi_a$ representing the superconducting condensate (i.e. the order parameter).
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- 3 self-interacting complex fields, \( \psi_a \) representing the superconducting condensate (i.e. the order parameter).
- charged under the same U(1) gauge field (\( \mathbf{A} \) is the vector potential of the magnetic field \( \nabla \times \mathbf{A} \)). \( e \) parametrizes the London penetration depth \( \lambda = \frac{1}{e \sqrt{\sum_a |\psi_a|^2}} \)
Multicomponent Ginzburg-Landau model

Three complex scalars $\psi_a = |\psi_a| \exp i \varphi_a$ are the SC condensate

$$F_{\text{3CGL}} = \frac{1}{2} (\nabla \times A)^2 + \frac{1}{2} \sum_{a=1}^{3} |(\nabla + ieA)\psi_a|^2 + (2\alpha_a + \beta_a |\psi_a|^2)|\psi_a|^2$$

$$- \sum_{b > a}^{3} \eta_{ab} |\psi_a||\psi_b| \cos \varphi_{ab} \quad \text{where} \quad \varphi_{ab} \equiv \varphi_b - \varphi_a .$$

- 3 self-interacting complex fields, $\psi_a$ representing the superconducting condensate (i.e. the order parameter).
- charged under the same U(1) gauge field ($A$ is the vector potential of the magnetic field $\nabla \times A$). $e$ parametrizes the London penetration depth $\lambda = \frac{1}{e \sqrt{\sum_a |\psi_a|^2}}$
- Josephson interband interaction. Couple all $\psi_a$'s. It breaks the global U(1)$^3$ symmetry of the potential down to U(1).
Elementary excitations in multi-component SCs

Basic excitations are fractional vortices

- *i.e.* only one condensate has $2\pi$ phase winding:

  \[ \Delta \varphi_1 = \oint \nabla \varphi_1 = 2\pi, \quad \Delta \varphi_2 = \oint \nabla \varphi_2 = 0, \quad \Delta \varphi_3 = \oint \nabla \varphi_3 = 0 \]

- each condensate carries a fraction of magnetic flux:

  \[ \Phi_a = \oint A \mathrm{d}\ell = \frac{|\psi_a|^2}{\sum_b |\psi_b|^2} \frac{1}{e} \oint \nabla \varphi_a = \frac{|\psi_a|^2}{\sum_b |\psi_b|^2} \Phi_0 \]

⇒ usually fractional vortices cannot be observed, since strongly bound
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  \]

- Only $\Delta \varphi_1 = \Delta \varphi_2 = \Delta \varphi_3 = 2\pi$ has finite energy (per unit length).
  \(
  \Rightarrow \text{Total} \text{ flux is quantized } \Phi \equiv \sum_a \Phi_a = \Phi_0
  \)

- Else, energy is logarithmically divergent if $\eta_{ab} = 0$
  or linearly divergent if $\eta_{ab} \neq 0$

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Elementary excitations in multi-component SCs

Basic excitations are fractional vortices

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- Else, energy is logarithmically divergent if \( \eta_{ab} = 0 \)
  or linearly divergent if \( \eta_{ab} \neq 0 \)

- This means fractional vortices are logarithmically bound if \( \eta_{ab} = 0 \)
  or linearly bound \( \eta_{ab} \neq 0 \)

\( \Rightarrow \) usually fractional vortices cannot be observed, since strongly bound
Frustration in three component systems

Back to full Ginzburg–Landau, look at the Josephson terms:

$$\mathcal{F}_{3\text{CGL}} = \cdots - \sum_{b>a}^{3} \eta_{ab} |\psi_a| |\psi_b| \cos \varphi_{ab}$$

where

$$\varphi_{ab} \equiv \varphi_b - \varphi_a.$$

- Each Josephson term **anti-locks** the phases ($\varphi_{ab} = \pi$) for $\eta_{ab} < 0$. 

$$\varphi_{1} \equiv 0, \quad \varphi_{2} = \frac{2\pi}{3}, \quad \varphi_{3} = -\frac{2\pi}{3}$$
Frustration in three component systems

Back to full Ginzburg–Landau, look at the Josephson terms:

\[ F_{3\text{CGL}} = \cdots - \sum_{b>a}^{3} \eta_{ab} |\psi_a| |\psi_b| \cos \varphi_{ab} \]

where \( \varphi_{ab} \equiv \varphi_b - \varphi_a \).

Each Josephson term anti-locks the phases (\( \varphi_{ab} = \pi \)) for \( \eta_{ab} < 0 \).

A simple example of frustration

- if \( \alpha_a = -1, \beta_a = 1 \) and \( \eta_{ab} = -1 \), one cannot have all phases differences \( \varphi_{ab} = \pi \).
- Then the ground state phases are (\( \varphi_1 \equiv 0 \))
  \[ \varphi_2 = \frac{2\pi}{3} \text{ and } \varphi_3 = \frac{-2\pi}{3} \]
  or
  \[ \varphi_2 = \frac{-2\pi}{3} \text{ and } \varphi_3 = \frac{2\pi}{3} \]

Here frustration \( \Rightarrow U(1) \times \mathbb{Z}_2 \) discrete symmetry of the ground state.
(spartaneously Broken Time Reversal Symmetry state)
Ground state in frustrated 3 component systems

To visualize, compute the minimal potential energy for a given phase configuration $\mathcal{F}_{pot}(\varphi_1 = 0, \varphi_2, \varphi_3)$

**Time Reversal Symmetric ground state**

- When all Josephson couplings lock the phases to zero *e.g.* all $\eta_{ab} > 0$
- A unique minimum $\Rightarrow U(1)$ symmetric ground state
Ground state in frustrated 3 component systems

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**Broken Time Reversal Symmetric ground state**

- When Josephson couplings are frustrated e.g. all $\eta_{ab} < 0$
- Two discrete minima
  $\Rightarrow U(1) \times \mathbb{Z}_2$ symmetric ground state

![Potential energy surface](image.png)
Ground state in frustrated 3 component systems

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Broken Time Reversal Symmetric ground state

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**U(1) \times \mathbb{Z}_2** symmetric ground state, \( \mathbb{CP}^2 \) Skyrmions

**Discrete symmetry \( \Rightarrow \) Domain-wall are natural solutions**

- Interpolate between inequivalent TRS broken state (\( \mathbb{Z}_2 \) states)
- Closed domain-wall is **unstable** to collapse because of its line tension.
**U(1) × Z₂ symmetric ground state, CP² Skyrmions**

**Discrete symmetry ⇒ Domain-wall are natural solutions**

- Interpolate between inequivalent TRS broken state (Z₂ states)
- Closed domain-wall is **unstable** to collapse because of its line tension.
- Domain wall has energetically unfavorable values of cos ϕ_{ab} ⇒ split integer vortices into fractional vortices
- More favorable phase gradients between vortices on the DW ⇒ vortices are confined on the domain-wall
U(1) \times \mathbb{Z}_2 \text{ symmetric ground state, } \mathbb{CP}^2 \text{ Skyrmions}

**Discrete symmetry }\Rightarrow\text{ Domain-wall are natural solutions**

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- More favorable phase gradients between vortices on the DW
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**Composite vortex/domain-wall solitons are in fact Chiral Skyrmion**

- If vortex interaction is \textit{repulsive} enough (repulsion between fractionnal vortices), captured vortices can stabilize the domain wall.

\textit{Chiral} refers to different TRSB states, i.e. different ground-state phases
Skyrmion’s structure \( (\eta_{ab} = -3; \alpha_a, \beta_a = 1; N = 5) \)

B

\[
\begin{align*}
|\psi_1| |\psi_2| \sin \varphi_{12} & \quad |\psi_1| |\psi_3| \sin \varphi_{13}
\end{align*}
\]

Chiral skyrmion’s features

- Ringlike Magnetic field, spread along the domain wall
- Phase difference \( \sin \varphi_{12} \) interpolate between the two inequivalent ground states \(-2\pi/3\) and \(2\pi/3\)

\[
\Rightarrow \quad \text{Domain-wall}
\]
Skyrmion’s structure ($\eta_{ab} = -3; \alpha_a, \beta_a = 1; N = 5$)

Chiral skyrmion’s features

- Singularity for each component do not superimpose
  ⇒ fractionalized vortices
- Even with Josephson interaction
Skyrmion’s structure \((\eta_{ab} = -3; \alpha_a, \beta_a = 1; N = 5)\)

| \(J_1|^2 || J_2|^2 || J_3|^2 |
|-----------------|----------------|----------------|

Chiral skyrmion’s features

- Nice current structure
- Supercurrents for each component do not superimpose
**Skyrmions topology – Chirality**

Topological invariant classifying the maps $\psi : \mathbb{R}^2 \rightarrow \mathbb{C}^3$

\[
Q(\psi) = \int_{\mathbb{R}^2} \frac{i\epsilon_{ij}}{2\pi|\psi|^4} \left[ |\psi|^2 \partial_i \psi^\dagger \partial_j \psi + \psi^\dagger \partial_i \psi \partial_j \psi^\dagger \psi \right] d^2x \in \mathbb{N},
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where $\psi^\dagger = (\psi_1^*, \psi_2^*, \psi_3^*)$ denotes the vector of the 3 complex fields ($\psi \neq 0$).
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where $\Psi^\dagger = (\psi_1^*, \psi_2^*, \psi_3^*)$ denotes the vector of the 3 complex fields ($\Psi \neq 0$).

The order of fractional vortices matters $\Rightarrow$ Chirality

- For a given vacuum (same outer phases), different ordering gives different skyrmions
  - Left unfavorable
  - Right favorable since inner phases are vacuum
Spin texture $S \equiv \frac{\psi^\dagger \sigma \psi}{\psi^\dagger \psi}$, of a $Q = 1$ skyrmion
Spin texture $S \equiv \frac{\psi^\dagger \sigma \psi}{\psi^\dagger \psi}$, of a $Q = 5$ skyrmion.
Plenty of solutions with unusual magnetic filed
Chiral skyrmions are (meta–)stable

They are very robust, here stable against 80% white noise
Physical properties of $\mathbb{CP}^2$ Chiral skyrmions

$\mathbb{CP}^2$ skyrmions are thermodynamically stable ($\frac{1}{2} H_{c1}^2 - |\mathcal{F}(\langle \psi_i \rangle, 0)| < 0$)

- are more energetic than vortices, but few percent only
  $\Rightarrow$ excited by thermal fluctuations or by quenching
  $\Rightarrow$ Metastable

- add $\sum_{b > a} \gamma_{ab} |\psi_a|^2 |\psi_b|^2$ to enhance vortex splitting
  $\Rightarrow$ Absolute minima
  $\Rightarrow$ trivially induced in applied field
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Plausible formation mechanisms suggested by numerics

- Relaxing an initially dense vortex cluster \([\text{Movie}]\)
- Trapping vortices on a domain wall \([\text{Movie}]\)
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Plausible formation mechanisms suggested by numerics

- Relaxing an initially dense vortex cluster [Movie]
- Trapping vortices on a domain wall [Movie]

Chiral skyrmions could be observable signature of TRSB states

They are very generic when the ground state exhibit $U(1) \times \mathbb{Z}_2$ symmetry.
Vortex pair as a $N = 2$, chiral skyrmion

- Magnetic field is peaked near vortices of the component with much more density and spread along the domain-wall
- Phase difference $\sin \varphi_{12}$ interpolate between the two inequivalent ground states $-\pi/2$ and $\pi/2$ ⇒ Domain-wall

Skyrmion’s features separation is several $\lambda$
Vortex pair as a $N = 2$, chiral skyrmion

Chiral skyrmion

Vortex pairs from Kalisky et al.

Skyrmion’s features

- however we still miss informations about the origin of vortex pairs
- also there is strong pinning in these materials

separation is several $\lambda$
Introduction

Vortex matter in phase frustrated three-component GL

Multicomponent Ginzburg-Landau and frustrated systems

Topological solitons in three component system \( \mathbb{CP}^2 \) Skyrmions

Summary

Chiral skyrmions could be an observable signature of TRSB states

- very exotic profile of magnetic field should be detected in scanning SQUID, scan Hall or magnetic force microscopy experiments
- Their counterparts exist also in two component systems where TRSB is broken by the underlying symmetries, and not by frustration. i.e. terms like \( \text{Re}(\psi_1^* \psi_2^2) \) for example in chiral \( p \)-wave superconductors, like \( \text{Sr}_2\text{RuO}_4 \). see JG and E. Babaev; Phys. Rev. B, 86, 060514(R) (2012)
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Still quite a lot open questions

- How can they influence phase transitions?
- Very complicated interactions, long-range repulsive, short-range attractive. Orientation dependent.
- Are there ordered structures, Skyrmion lattices?
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Topological solitons in three component system ($\mathbb{C}P^2$ Skyrmions)

Thank you for your attention!

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More Chiral skyrmions

\( \text{CP}^2 \) Skyrmions are very generic and robust

Here, one of the component has much more density than the others. The solution carries \( N = 3 \) flux quanta.
More Chiral skyrmions

\[ \sin \varphi_{12} \quad \sin \varphi_{13} \]

\( \mathbb{CP}^2 \) Skyrmions are very generic and robust here, one of the component has much more density than the others, the solution carries \( N = 5 \) flux quanta.
More Chiral skyrmions

$\mathbb{C}P^2$ Skyrmions are very generic and robust

another $\mathbb{C}P^2$ soliton with $N = 3$
More Chiral skyrmions

$\mathbb{CP}^2$ Skyrmions are very generic and robust

another $\mathbb{CP}^2$ soliton with $N = 5$
Multi chiral skyrmions

(Alternating) multiple Skyrmions are also easily constructed

Two chiral solitons, a $N = 7$ inside $N = 13$
Multi chiral skyrmions

(Alternating) multiple Skyrmions are also easily constructed

Three chiral solitons, a $N = 6$ inside $N = 12$ himself in a $N = 18$
Skyrmions are generic solutions of $\text{U}(1) \times \mathbb{Z}_2$ BTRS

Adding a term $\sum_{b>a}^{3} \gamma_{ab} |\psi_a|^2 |\psi_b|^2$ enhances vortex splitting

- Skyrmions can be less energetic than vortices $\Rightarrow$ ground state
- They are thermodynamically stable when $\frac{1}{2} H_{c1}^2 - |\mathcal{F}(\langle \psi_i \rangle, 0)| < 0$

Here fixed $e = 0.3$ and different curves are different $\gamma_{ab}$.
For large enough $\gamma_{ab}$, Skyrmions are preferred over vortices
Skyrmions are generic solutions of $U(1) \times \mathbb{Z}_2$ BTRS

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