Liquid Crystal Director Models with Coupled Electric Fields

Chuck Gartland

Department of Mathematical Sciences

Kent State University

Thanks:

John Toland (Isaac Newton Institute)

John Ball, David Chillingworth, Mikhail Osipov, Peter Palffy-Muhoray, Mark Warner (Programme)

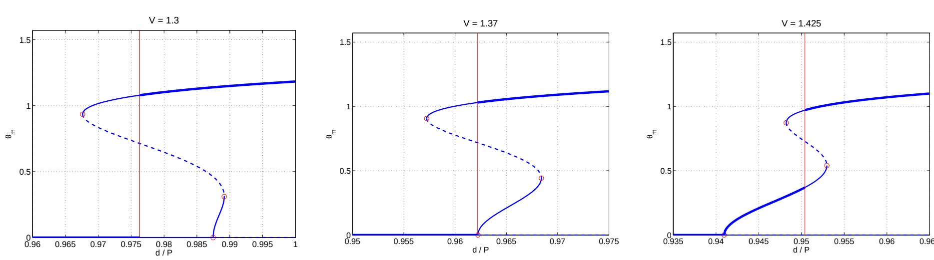
Mikhail Osipov, Valeriy Slastikov, Mike Allen, Claudio Zannoni (Workshop)

Support:

National Science Foundation grant DMS-1211597

Motivation

- Numerical modeling of devices and experiments: equilibrium orientational properties, coupled electric fields, Oseen-Frank.
- Numerical bifurcation and phase analysis:
  - discretized free energy (finite differences or finite elements or . . .): $f = f(\mathbf{n}, \boldsymbol{\varphi})$, $\mathbf{n} = \{\boldsymbol{n}_i\}$ = directors, $\boldsymbol{\varphi} = \{\varphi_i\}$ = electric potential
  - Lagrangian: $L(\mathbf{n}, \boldsymbol{\varphi}, \boldsymbol{\lambda}) = f(\mathbf{n}, \boldsymbol{\varphi}) + \sum \lambda_i \left(|\boldsymbol{n}_i|^2 - 1 \right)$
  - paths of equilibria ($\nabla L = \mathbf{0}$) via parameter continuation
  - bifurcation and branch switching
  - local stability assessment



Questions and Issues

- Characterization of local stability.
- Extension from discrete setting to continuum setting.
- Effect of $\boldsymbol{n}$ - $\boldsymbol{E}$ coupling on electric-field-induced instabilities, e.g., Fréedericksz- -transition thresholds.
- Anomalous behavior of the electric-field bend-Fréedericksz transition.

Free Energy

Typical Situation: capacitor, constant voltage, $\boldsymbol{E} = -\nabla\varphi =$ local electric field

$$\mathcal{F}[\boldsymbol{n},\varphi] = \int_\Omega W(\boldsymbol{n},\nabla\boldsymbol{n},\nabla\varphi)\,dV + \int_{\Gamma_1} W_{\mathsf{b}}(\boldsymbol{n})\,dS$$

$$W(\boldsymbol{n},\nabla\boldsymbol{n},\nabla\varphi) = W_{\mathsf{e}}(\boldsymbol{n},\nabla\boldsymbol{n}) - W_{\mathsf{E}}(\boldsymbol{n},\nabla\varphi)$$

Distortional Elasticity: e.g., $W_{\mathsf{e}} = \frac{1}{2}K|\nabla\boldsymbol{n}|^2$, $\boldsymbol{n} =$ "director field", $|\boldsymbol{n}| = 1$

Electrostatic Energy: $W_{\mathsf{E}} = \frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E} = \frac{1}{2}\boldsymbol{\varepsilon}(\boldsymbol{n})\nabla\varphi\cdot\nabla\varphi$

$$[\boldsymbol{\varepsilon}] = \varepsilon_0 \begin{bmatrix} \varepsilon_\perp & & \\ & \varepsilon_\perp & \\ & & \varepsilon_\parallel \end{bmatrix}_{\boldsymbol{l},\boldsymbol{m},\boldsymbol{n}} \quad \leftrightarrow \quad \boldsymbol{\varepsilon} = \varepsilon_0\big[\varepsilon_\perp\mathbf{I} + \varepsilon_{\mathsf{a}}\boldsymbol{n}\otimes\boldsymbol{n}\big], \quad \varepsilon_{\mathsf{a}} := \varepsilon_\parallel - \varepsilon_\perp$$

Surface Anchoring Potential: e.g., $W_{\mathsf{b}} = -\frac{1}{2}W_0(\boldsymbol{n}\cdot\boldsymbol{\nu})^2$, $\boldsymbol{\nu} =$ unit normal

$\partial\Omega = \Gamma_0 \cup \Gamma_1$, Γ_0 "strong anchoring" $(\boldsymbol{n} = \boldsymbol{n}_{\mathsf{b}})$, Γ_1 "weak anchoring" (W_{b})

General Form:

$$\mathcal{F}[\boldsymbol{n},\varphi] = \int_\Omega \Big[W_{\mathsf{e}}(\boldsymbol{n},\nabla\boldsymbol{n}) - \frac{1}{2}\boldsymbol{\varepsilon}(\boldsymbol{n})\nabla\varphi\cdot\nabla\varphi \Big] + \int_{\Gamma_1} W_{\mathsf{b}}(\boldsymbol{n})$$

Prototype:

$$\begin{aligned}\mathcal{F}[\boldsymbol{n},\varphi] &= \frac{1}{2}\int_\Omega \big[K|\nabla\boldsymbol{n}|^2 - \boldsymbol{\varepsilon}(\boldsymbol{n})\nabla\varphi\cdot\nabla\varphi\big] \\ &= \frac{1}{2}\int_\Omega \Big\{ K|\nabla\boldsymbol{n}|^2 - \varepsilon_0\big[\varepsilon_\perp|\nabla\varphi|^2 + \varepsilon_{\mathsf{a}}(\nabla\varphi\cdot\boldsymbol{n})^2\big]\Big\}\end{aligned}$$

Influence:

$$\varepsilon_{\mathsf{a}} > 0 \;\Rightarrow\; \boldsymbol{n} \parallel \boldsymbol{E} \qquad \text{vs} \qquad \varepsilon_{\mathsf{a}} < 0 \;\Rightarrow\; \boldsymbol{n} \perp \boldsymbol{E}$$

Boundary Conditions: various possibilities . . . for simplicity

$$\boldsymbol{n} = \boldsymbol{n}_{\mathsf{b}} \text{ on } \Gamma_0, \qquad \varphi = \varphi_{\mathsf{b}} \text{ on } \partial\Omega$$

Equilibria

Globally Stable Phase: $\boldsymbol{n}_0 \in \mathcal{N}_{\mathsf{b}}$, $\varphi_0 \in \Phi_{\mathsf{b}}$ (admissible fields) such that

$$\mathcal{F}[\boldsymbol{n}_0,\varphi_0] = \min_{\boldsymbol{n}\in\mathcal{N}_{\mathsf{b}}} \max_{\varphi\in\Phi_{\mathsf{b}}} \mathcal{F}[\boldsymbol{n},\varphi]$$

equivalently

$$\min_{\boldsymbol{n}\in\mathcal{N}_{\mathsf{b}}} \mathcal{F}[\boldsymbol{n},\varphi], \text{ subject to } \operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n})\nabla\varphi\big) = \sum_{i,j=1}^{3} \frac{\partial}{\partial x_i}\Big(\varepsilon_{ij}\frac{\partial\varphi}{\partial x_j}\Big) = 0$$

where

$$\begin{aligned}\mathcal{N}_{\mathsf{b}} &= \big\{\boldsymbol{n}\,\big|\,|\boldsymbol{n}| = 1 \text{ in } \Omega,\; \boldsymbol{n} = \boldsymbol{n}_{\mathsf{b}} \text{ on } \Gamma_0\big\} \\ \Phi_{\mathsf{b}} &= \big\{\varphi\,\big|\,\varphi = \varphi_{\mathsf{b}} \text{ on } \partial\Omega\big\}\end{aligned}$$

Point: intrinsic *minimax* nature $\leftrightarrow$ PDE-constrained minimization

1st-Order Conditions: $(\boldsymbol{n}_0, \varphi_0) \in \mathcal{N}_{\mathsf{b}} \times \Phi_{\mathsf{b}}$ equilibrium pair $\Rightarrow$

weak form #1: $\mathcal{U}_0 := \{\boldsymbol{u} \,|\, \boldsymbol{n}_0 \cdot \boldsymbol{u} = 0,\ \boldsymbol{u} = \boldsymbol{0} \text{ on } \Gamma_0\},\ \Phi_0 := \{\psi \,|\, \psi = 0 \text{ on } \partial\Omega\}$

$$\delta_{\boldsymbol{n}} \mathcal{F}[\boldsymbol{n}_0, \varphi_0](\boldsymbol{u}) = 0,\ \forall \boldsymbol{u} \in \mathcal{U}_0$$

$$\delta_{\varphi} \mathcal{F}[\boldsymbol{n}_0, \varphi_0](\psi) = 0,\ \forall \psi \in \Phi_0$$

weak form #2: $\lambda_0,\ \mu_0$ Lagrange multiplier fields, $\mathcal{V}_0 := \{\boldsymbol{v} \,|\, \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma_0\}$

$$\delta_{\boldsymbol{n}} \mathcal{F}[\boldsymbol{n}_0, \varphi_0](\boldsymbol{v}) = \int_\Omega \lambda_0\, \boldsymbol{n}_0 \cdot \boldsymbol{v} + \int_{\Gamma_1} \mu_0\, \boldsymbol{n}_0 \cdot \boldsymbol{v},\ \forall \boldsymbol{v} \in \mathcal{V}_0$$

$$\int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}_0) \nabla \varphi_0 \cdot \nabla \psi = 0,\ \forall \psi \in \Phi_0$$

strong form:

$$-\operatorname{div}\left(\frac{\partial W_{\mathsf{e}}}{\partial \nabla \boldsymbol{n}}\right) + \frac{\partial W_{\mathsf{e}}}{\partial \boldsymbol{n}} = \lambda_0 \boldsymbol{n}_0 + \varepsilon_0 \varepsilon_{\mathsf{a}} \left(\nabla \varphi_0 \cdot \boldsymbol{n}_0\right) \nabla \varphi_0 \ \text{ in } \Omega$$

$$\boldsymbol{n}_0 = \boldsymbol{n}_{\mathsf{b}} \text{ on } \Gamma_0, \quad \left(\frac{\partial W_{\mathsf{e}}}{\partial \nabla \boldsymbol{n}}\right) \boldsymbol{\nu} + \frac{\partial W_{\mathsf{b}}}{\partial \boldsymbol{n}} = \mu_0 \boldsymbol{n}_0 \ \text{ on } \Gamma_1$$

$$\operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n}_0) \nabla \varphi_0\big) = 0 \ \text{ in } \Omega, \quad \varphi_0 = \varphi_{\mathsf{b}} \ \text{ on } \partial\Omega$$

Question: 2nd-order conditions? local stability?

Deflated Free Energy

$$\widetilde{\mathcal{F}}[\boldsymbol{n}] := \max_{\varphi \in \Phi_{\mathsf{b}}} \mathcal{F}[\boldsymbol{n}, \varphi] = \mathcal{F}[\boldsymbol{n}, T(\boldsymbol{n})], \quad \text{where } \varphi = T(\boldsymbol{n}) = \text{solution to}$$

$$\delta_{\varphi} \mathcal{F}[\boldsymbol{n}, \varphi] = 0 \ \Leftrightarrow \ \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}) \nabla \varphi \cdot \nabla \psi = 0,\ \varphi \in \Phi_{\mathsf{b}},\ \forall \psi \in \Phi_0$$

H^1**-ellipticity:**

$$\varepsilon_0 \min\{\varepsilon_\perp, \varepsilon_\parallel\} \int_\Omega |\nabla \varphi|^2 \le \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}) \nabla \varphi \cdot \nabla \varphi \le \varepsilon_0 \max\{\varepsilon_\perp, \varepsilon_\parallel\} \int_\Omega |\nabla \varphi|^2$$

Derivative: $\psi = DT(\boldsymbol{n}_0) \boldsymbol{v} =$ solution to: $\psi \in \Phi_0$,

$$\int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}_0) \nabla \psi \cdot \nabla \chi = \int_\Omega \boldsymbol{d}_0 \cdot \nabla \chi,\ \forall \chi \in \Phi_0 \ \Leftrightarrow \ \operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n}_0) \nabla \psi\big) = \operatorname{div} \boldsymbol{d}_0$$

$$\boxed{\boldsymbol{d}_0 := \varepsilon_0 \varepsilon_{\mathsf{a}} (\boldsymbol{n}_0 \otimes \boldsymbol{v} + \boldsymbol{v} \otimes \boldsymbol{n}_0) \boldsymbol{E}_0},\ \boldsymbol{E}_0 = -\nabla \varphi_0,\ \varphi_0 = T(\boldsymbol{n}_0)$$

Interpretation:

$\boldsymbol{d}_0 =$ 1st-order change in $\boldsymbol{D}_0 = \boldsymbol{\varepsilon}(\boldsymbol{n}_0) \boldsymbol{E}_0$ associated with $\boldsymbol{n}_0 \mapsto \boldsymbol{n}_0 + \boldsymbol{v}$

Alternately: $\operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n}_0)\nabla\varphi_0\big) = 0, \ \ \boldsymbol{\varepsilon}(\boldsymbol{n}) = \varepsilon_0\big[\varepsilon_\perp \mathbf{I} + \varepsilon_{\mathsf{a}}(\boldsymbol{n}\otimes\boldsymbol{n})\big]$

$$\operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n}_0+\boldsymbol{v})\nabla(\varphi_0+\psi)\big) = 0 + \operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n}_0)\nabla\psi\big) - \operatorname{div}\boldsymbol{d}_0 + \cdots$$

Alternately: $\delta^2\mathcal{F} = \delta^2_{\boldsymbol{n}\boldsymbol{n}}\mathcal{F} + 2\delta^2_{\boldsymbol{n}\varphi}\mathcal{F} + \delta^2_{\varphi\varphi}\mathcal{F}$

$$\delta^2\mathcal{F}[\boldsymbol{n}_0,\varphi_0]\big((\boldsymbol{v},\psi),(\boldsymbol{v},\psi)\big) = \delta^2_{\boldsymbol{n}\boldsymbol{n}}\mathcal{F}[\boldsymbol{n}_0,\varphi_0](\boldsymbol{v},\boldsymbol{v}) + 2\int_\Omega \boldsymbol{d}_0\cdot\nabla\psi - \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}_0)\nabla\psi\cdot\nabla\psi$$

Globally Stable Phase: $\widetilde{\mathcal{F}}$ is coercive! ... $\boldsymbol{n}_0 \in \mathcal{N}_{\mathsf{b}}$ such that

$$\widetilde{\mathcal{F}}[\boldsymbol{n}_0] = \min_{\boldsymbol{n}\in\mathcal{N}_{\mathsf{b}}} \widetilde{\mathcal{F}}[\boldsymbol{n}] \ \ \leftrightarrow$$ equilibrium pair $(\boldsymbol{n}_0,\varphi_0)$ of "Least Free Energy"

1st-Order and 2nd-Order Conditions (in terms of $\widetilde{\mathcal{F}}$):

$$\delta\widetilde{\mathcal{F}}[\boldsymbol{n}_0](\boldsymbol{v}) = \int_\Omega \lambda_0\,\boldsymbol{n}_0\cdot\boldsymbol{v} + \int_{\Gamma_1}\mu_0\,\boldsymbol{n}_0\cdot\boldsymbol{v}, \ \ \forall\boldsymbol{v}\in\mathcal{V}_0$$

$$\delta^2\widetilde{\mathcal{F}}[\boldsymbol{n}_0](\boldsymbol{u},\boldsymbol{u}) - \int_\Omega \lambda_0|\boldsymbol{u}|^2 - \int_{\Gamma_1}\mu_0|\boldsymbol{u}|^2 \geq 0, \ \ \forall\boldsymbol{u}\in\mathcal{U}_0$$

Calculus: relate $\delta\widetilde{\mathcal{F}}, \delta^2\widetilde{\mathcal{F}}$ to $\delta\mathcal{F}, \delta^2\mathcal{F}$ ...

$$\delta\widetilde{\mathcal{F}}[\boldsymbol{n}_0](\boldsymbol{v}) = \delta_{\boldsymbol{n}}\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)](\boldsymbol{v}) + \delta_\varphi\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)](DT(\boldsymbol{n}_0)\boldsymbol{v})$$

$$= \delta_{\boldsymbol{n}}\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)](\boldsymbol{v}), \ \text{ using } \delta_\varphi\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)] = 0$$

$$\delta^2\widetilde{\mathcal{F}}[\boldsymbol{n}_0](\boldsymbol{v},\boldsymbol{v}) = \delta^2_{\boldsymbol{n}\boldsymbol{n}}\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)](\boldsymbol{v},\boldsymbol{v}) + \delta^2_{\boldsymbol{n}\varphi}\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)](\boldsymbol{v},DT(\boldsymbol{n}_0)\boldsymbol{v})$$

$$= \delta^2_{\boldsymbol{n}\boldsymbol{n}}\mathcal{F}[\boldsymbol{n}_0,T(\boldsymbol{n}_0)](\boldsymbol{v},\boldsymbol{v}) + \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}_0)\nabla\psi\cdot\nabla\psi, \ \ \psi = DT(\boldsymbol{n}_0)\boldsymbol{v}$$

using

$$\delta^2_{\boldsymbol{n}\varphi}\mathcal{F}[\boldsymbol{n}_0,\varphi_0](\boldsymbol{v},\psi) = \int_\Omega \boldsymbol{d}_0\cdot\nabla\psi$$

and

$$\psi = DT(\boldsymbol{n}_0)\boldsymbol{v} \ \ \Rightarrow \ \ \int_\Omega \boldsymbol{d}_0\cdot\nabla\psi = \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}_0)\nabla\psi\cdot\nabla\psi$$

Conclusion (in terms of $\mathcal{F}$): $(\boldsymbol{n}_0, \varphi_0) \in \mathcal{N}_{\mathrm{b}} \times \Phi_{\mathrm{b}}$ locally stable equilibrium pair $\Rightarrow$

$$\delta^2_{\boldsymbol{nn}} \mathcal{F}[\boldsymbol{n}_0, \varphi_0](\boldsymbol{u}, \boldsymbol{u}) + \int_\Omega \boldsymbol{\varepsilon}(\boldsymbol{n}_0) \nabla\psi \cdot \nabla\psi - \int_\Omega \lambda_0 |\boldsymbol{u}|^2 - \int_{\Gamma_1} \mu_0 |\boldsymbol{u}|^2 \geq 0,$$

$$\forall \boldsymbol{u} \in \mathcal{U}_0, \text{ with } \psi = DT(\boldsymbol{n}_0)\boldsymbol{u}$$

Alternately:

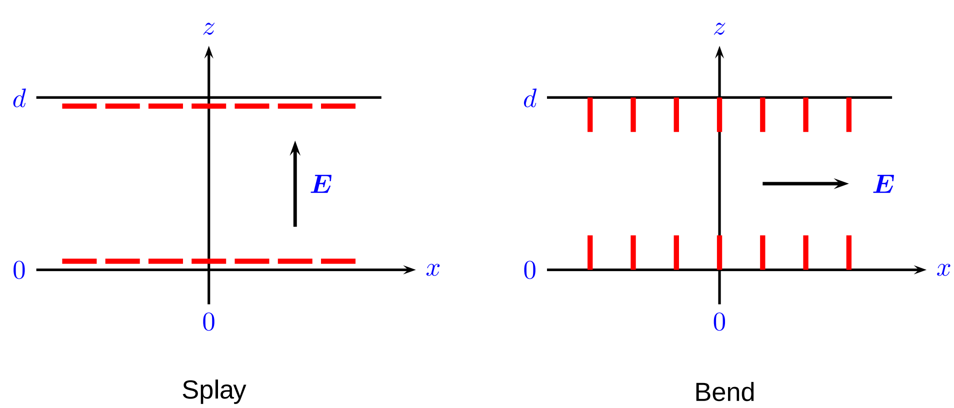
$$\max_{\psi \in \Phi_0} \delta^2 \mathcal{F}[\boldsymbol{n}_0, \varphi_0]\big((\boldsymbol{u}, \psi), (\boldsymbol{u}, \psi)\big) - \int_\Omega \lambda_0 |\boldsymbol{u}|^2 - \int_{\Gamma_1} \mu_0 |\boldsymbol{u}|^2 \geq 0, \ \forall \boldsymbol{u} \in \mathcal{U}_0$$

Points: recall strong form ψ: $\operatorname{div}\big(\boldsymbol{\varepsilon}(\boldsymbol{n}_0)\nabla\psi\big) = \operatorname{div} \boldsymbol{d}_0$ in Ω, $\psi = 0$ on $\partial\Omega$

1. $\operatorname{div} \boldsymbol{d}_0 = 0 \ \Rightarrow \ \psi = 0$ $\Rightarrow$ coupling has *no effect* on local stability of an equilibrium phase.
2. $\operatorname{div} \boldsymbol{d}_0 \neq 0 \ \Rightarrow \ \psi \neq 0$ $\Rightarrow$ coupling is *stabilizing* (e.g., can only *elevate* an instability threshold).
3. The φ_0 variation ψ is slaved to the $\boldsymbol{n}_0$ variation $\boldsymbol{u}$, just as φ_0 is slaved to $\boldsymbol{n}_0$.

Application

Electric-Field Fréedericksz Transitions: "splay geometry" vs "bend geometry"



Splay: references: Deuling (MCLC, 1972); Iain Stewart book (2004)

$$\boldsymbol{n}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \;\Rightarrow\; \boldsymbol{u} = \begin{bmatrix} 0 \\ 0 \\ w(z) \end{bmatrix}, \quad \boldsymbol{E}_0 = \begin{bmatrix} 0 \\ 0 \\ E_0 \end{bmatrix}$$

$$\Rightarrow \quad \boldsymbol{d}_0 = \varepsilon_0 \varepsilon_{\mathsf{a}} (\boldsymbol{n}_0 \otimes \boldsymbol{u} + \boldsymbol{u} \otimes \boldsymbol{n}_0) \boldsymbol{E}_0 = \varepsilon_0 \varepsilon_{\mathsf{a}} E_0 \begin{bmatrix} w(z) \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad \operatorname{div} \boldsymbol{d}_0 = 0 \quad \Rightarrow$ no effect on stability

threshold: $E_{\mathsf{c}}^2 = \dfrac{K_1}{\varepsilon_0 \varepsilon_{\mathsf{a}}} \dfrac{\pi^2}{d^2}$ vs magnetic-field case $H_{\mathsf{c}}^2 = \dfrac{K_1}{\mu_0 \Delta\chi} \dfrac{\pi^2}{d^2}$

Bend:

$$\boldsymbol{n}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \;\Rightarrow\; \boldsymbol{u} = \begin{bmatrix} u(z) \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{E}_0 = \begin{bmatrix} E_0 \\ 0 \\ 0 \end{bmatrix}$$

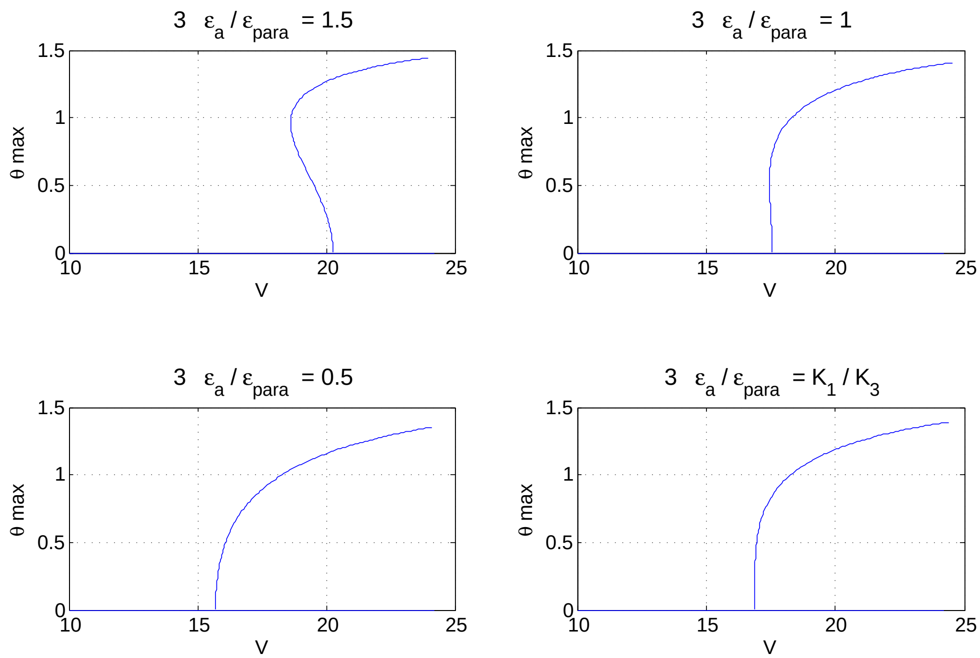
$$\Rightarrow \quad \boldsymbol{d}_0 = \varepsilon_0 \varepsilon_{\mathsf{a}} E_0 \begin{bmatrix} 0 \\ 0 \\ u(z) \end{bmatrix} \quad \Rightarrow \quad \operatorname{div} \boldsymbol{d}_0 = \varepsilon_0 \varepsilon_{\mathsf{a}} E_0 u'(z) \neq 0$$

elevated threshold:

$$E_{\mathsf{c}}^2 (\text{ignore coupling}) = \frac{K_3}{\varepsilon_0 \varepsilon_{\mathsf{a}}} \frac{\pi^2}{d^2} \quad \text{vs} \quad E_{\mathsf{c}}^2 (\text{with coupling}) = \frac{\varepsilon_\parallel}{\varepsilon_\perp} \frac{K_3}{\varepsilon_0 \varepsilon_{\mathsf{a}}} \frac{\pi^2}{d^2}$$

Remarks:

1. Effect is *not* small: for 5CB, $\varepsilon_\parallel = 18.5,\ \varepsilon_\perp = 7 \;\Rightarrow\; 63\%$ higher voltage!
2. Transition can be *1st order*: $K_1/K_3 < 3\varepsilon_{\mathsf{a}}/\varepsilon_\parallel \approx 1.86$, for 5CB
3. Reference: Arakelyan, Karayan, and Chilingaryan, Dokl. Akad. Nauk SSSR, 1984.



Reference: Gregory P. Richards, Masters Thesis, Kent State University, 2006

($K_1/K_3 = 3.6/4.3 \doteq 0.84$)

Comparison with Experiment

??? MAKE A FIGURE HERE WITH FRISKEN-PALFFY DATA VS BIFURCATION DIAGRAM FOR THOSE MATERIAL PARAMETERS (JOHN BALL QUESTION) ???

Reference: Frisken and Palffy-Muhoray, *Electric-field-induced twist and bend Fréedericksz transitions in nematic liquid crystals*, PRA **39** (1989).

Conclusions

- Local stability assessment complicated by pointwise constraint and minimax nature.
- One-sided nature of the coupling: can only *elevate* an instability threshold.
- Instability threshold elevation: $\delta\varphi_0 = O\big(|\boldsymbol{\delta n}_0|\big)$ vs $O\big(|\boldsymbol{\delta n}_0|^2\big)$?
- Similar effects in ferroelectrics, flexoelectric polarization terms, . . .
- Numerical implementation: $\lambda_{\min}\big(Z^T\big(A + DC^{-1}D^T\big)Z\big) \geq 0$
- Additional references:
  - Rosso, Virga, and Kralj, *Local elastic stability for nematic liquid crystals*, PRE **70** (2004).
  - Bevilacqua and Napoli, *Re-examination of the Helfrich-Hurault effect in smectic-A liquid crystals*, PRE **72** (2005).