Quantitative Modelling of Market Booms and Crashes

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"October. This is one of the peculiarly dangerous months to speculate in stocks. The others are July, January, September, April, November, May, March, June, December, August and February.” - M. Twain
Outline

Models that explain market crashes

Multiple Equilibria Modelling in Continuous Time

Market microstructure models

Alternative models

Main results
Models that explain market crashes

- Temporary reduction in liquidity
  - Market price might plummet due to a temporary reduction in liquidity
- Multiple equilibria
  - Several price levels exist and a market crash might occur for no fundamental reason
Models that explain market crashes

- Bursting bubble
  - All market participants realise an asset price is greater than its fundamental value
  - They keep buying that asset since they believe others do not know that it is overpriced
  - At some point the bubble bursts and market crashes
Models that explain market crashes

- Lumpy information aggregation
  - The overpricing issue is not a common knowledge among the market participants
  - At some point an additional relevant information is revealed
  - Combining that with the past price dynamics, less informed traders suddenly realise that this overpricing exists and the price sharply declines

- Large bets and market microstructure invariance
  - Too fast executions of large bets might lead to market crashes
Outline

Models that explain market crashes

**Multiple Equilibria Modelling in Continuous Time**

Market microstructure models

Alternative models

Main results
Model multiple equilibria based on the market microstructure framework and demonstrate how market prices move from one regime into another.
Multiple Equilibria Modelling in Continuous Time

- Model multiple equilibria based on the market microstructure framework and demonstrate how market prices move from one regime into another.

- As a consequence of this, a multiple jump structure can be obtained with both possible booms and crashes, which are defined as points of discontinuity of the stock price process.
Market microstructure models

- Examples of models:
  - Endogenous switching
  - Exogenous shocks
  - Stochastic number of dynamic hedgers
Market microstructure models

- Examples of models:
  - Endogenous switching
  - Exogenous shocks
  - Stochastic number of dynamic hedgers

- The difference between the models is in mechanisms for determining how the market price moves from one regime into another
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Market microstructure framework

- We work on a filtered stochastic base $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ satisfying the usual conditions.

- Time horizon is $[0, T]$ and trading takes place continuously.

- For the sake of simplicity, it is supposed that there is no information asymmetry.

- There are two underlying assets in the economy: a risk-free bond and a risky stock.
Market microstructure framework

- The risk-free bond is in perfectly elastic supply and grows at net return $r > 0$: one unit invested at time $t$ returns $e^{r\Delta t}$ units at time $t + \Delta t$, $0 \leq t < t + \Delta t \leq T$

- The risky stock is assumed to be in zero net supply
Market microstructure framework

- In making their decisions, agents use their wealth \((W_s, 0 \leq s \leq t < T)\), the stock price process \((P_s, 0 \leq s \leq t < T)\) and an auxiliary process \((p_u, t \leq u \leq T)\) such that

\[
p_u = P_t + \alpha_1 \times \beta_{u-t} + \alpha_2 \times (u - t),
\]

where \(\beta\) is a standard Brownian motion that starts at 0, \(\alpha_1 > 0\) and \(\alpha_2 \in \mathbb{R}\).

- This process \((p_u, t \leq u \leq T)\) approximates the future dynamics of the stock price \((P_u, t \leq u \leq T)\).
Agents

- **Rational investors**
  - Maximize expected CARA with coefficient $a > 0$ utility of their wealth at $t + \Delta t$
  - Their component of demand: $w^R \times \frac{a(\alpha_2 - rP_t)}{\alpha_1^2}$

- **Dynamic hedgers**
  - Replicate European calls with maturity $T$
  - Their component of demand:
    
    $$w^D \times \int_{-\infty}^{\infty} \Phi\left(\frac{P_t - Ke^{-r(T-t)}}{\Sigma(t)}\right) \frac{1}{\sqrt{2\pi}\sigma^2_\kappa} e^{-\frac{(K-\kappa)^2}{2\sigma^2_\kappa}} dK$$

- **Noise traders**
  - Their component of demand:
    
    $$w^N \times (\mu^N + \sigma^N B_t)$$
The market clearing condition states that the total demand should be equal to 0

Pricing equation is given by

\[ h(t, P_t) = B_t, \]

where \( B_t \) is a Brownian motion and function \( h(t, x) \) is deterministic and smooth.
Market microstructure framework

If the number of dynamic hedgers is large enough, then we might have multiple equilibria:

The number of dynamic hedgers is small

\[ h(t, x) \]

The number of dynamic hedgers is large

\[ h(t, x) \]
Endogenous switching model

Brownian motion

State process

Stock price
Exogenous shocks model

Brownian motion

State process

Stock price
Stochastic number of dynamic hedgers model

- Brownian motion $B_t$
- Number of Hedgers $w^D_t$
- State Process $S_t$
- Stock Price $P_t$
Outline

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Main results
Motivation

- Eliminate the possibility of negative prices
- Actual price is not an arithmetic Brownian Motion
- Find distributions in a closed form
- Overcome the jump structure problem
Alternative framework

- **Simple jump structure model**

  - Eliminates the possibility of negative prices
  - Does not have an Arithmetic Brownian Motion approximation
  - We can find distributions in a closed form

- **Markov chain jump structure model**
  - On top of all of this, the model overcomes the jump structure problem
Outline

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Main results
Main results

- For all the models, we
  - prove that the stock price is a càdlàg semimartingale process
  - find conditional distributions for the time of the next jump, the type of the next jump and the size of the next jump, given the public information available to market participants
  - discuss the problem of model parameter estimation
  - conduct a number of numerical studies