Decomposing Stochastic Unit Commitment Problems

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Agenda

- Motivation
- Multistage Stochastic Unit Commitment Formulation
- Decomposition Schemes
- A Column Generation Algorithm
- Practical Issues
- Numerical Results
- Conclusion
Motivation - Wind Power, Price Uncertainty

- Short term: Supplies from volatile renewable sources. EU 2011: 94GW, estimated 2020: 230GW → increased predictability problem
- Medium term: price uncertainty of fossil fuels
- Account for increased uncertainty in conventional scheduling: Unit Commitment
Deterministic Unit Commitment Problem

- Minimum cost on/off (binary) and power output decisions (24h)
- Constraints: load balance, spinning reserve, generator bounds, up/downtimes, ramp rates
- Large MI(Q)P, solve by B&B
Stochastic Unit Commitment Problem

- Multiple scenarios with different residual loads / different prices
- Tree structure: scenario information revealed over time
- Problem: size of the model increases quickly in the number of scenarios

![Graph showing load vs. generation over time with multiple scenarios]

Load = Generation

Time 0 2 4 6 8 10 12 14 16 18 20 22 0

Decomposing Stochastic Unit Commitment Problems

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Decomposition Schemes

- by scenarios
- by stages
- by generating units

Tree with four wind / price scenarios.
Decomposition Schemes

► by scenarios
► by stages
► by generating units

The stages can be interpreted as scenario bundles.
Decomposition Schemes

- by scenarios
- by stages
- by generating units

Duplicate the variables and negotiate compliance with the bundles.
Decomposition Schemes

- by scenarios
- by stages
- by generating units

Solve scenario problems independently.
Decomposition Schemes

- by scenarios
- by stages
- by generating units

Tree with four wind / price scenarios.
Decomposition Schemes

- by scenarios
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Uncouple the stages, negotiate the system state on the boundary.
Decomposition Schemes

- by scenarios
- by stages
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Solve the stage subproblems independently.
Decomposition Schemes

- by scenarios
- by stages
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Tree with four wind / price scenarios.
Decomposition Schemes

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A single generator problem. Negotiate its contribution to demand and reserve.
Decomposition Schemes

- by scenarios
- by stages
- by generating units

Duplicate stochastic trees for each generator.
Decomposition Schemes

- by scenarios
- by stages
- by generating units

Solve the generator subproblems independently.
## Decomposition Schemes: Summary

- **by scenarios**
  - +: gives a tight lower bound for the optimal integer solution, converges quickly
  - -: relatively large subproblems

- **by stages**
  - +: gives a tight lower bound for the optimal integer solution
  - -: converges slowly, doesn’t scale well if the problem size is increased

- **by generating units**
  - +: small, easy to solve subproblems
  - -: gives a lower bound which is not much better than the LP relaxation
Stochastic Unit Commitment Problem

\[
\min_{\alpha, \gamma, \eta, p} \sum_{s \in S} Pr_s \sum_{t=1}^{T} \sum_{g \in G} f_g (\alpha_{gts}, \gamma_{gts}, \eta_{gts}, p_{gts})
\]

s.t.

\((\alpha_s, \gamma_s, \eta_s, p_s) \in X_s, \forall s \in S\)

\(\alpha_{gts} = \bar{\alpha}_{gbt}, \forall g \in G, b \in B, t = t_{b1}, \ldots, t_{b2}, s \in S_b\)

Notation

- **S** - scenarios
- **B** - bundles
- **S_b** - scenarios in bundle **b**
- **X_s** - technical constraints
- **t_{b1}** - start time of **b**
- **t_{b2}** - end time of **b**
- **Pr_s** - probability of scenarios
- **\bar{\alpha}** - common commitments in a bundle
**Stochastic Unit Commitment Problem**

\[
\min_{\alpha, \gamma, \eta, p} \sum_{s \in S} \Pr_s \sum_{t=1}^{T} \sum_{g \in G} f_g (\alpha_{gts}, \gamma_{gts}, \eta_{gts}, p_{gts})
\]

s.t.

\[
(\alpha_s, \gamma_s, \eta_s, p_s) \in \mathcal{X}_s, \ \forall s \in S
\]

\[
\alpha_{gts} = \bar{\alpha}_{gbt}, \ \forall g \in G, b \in B, \ t = t_{b1}, \ldots, t_{b2}, s \in S_b
\]

**Relaxing Nonanticipativity \rightarrow Separable Subproblem**

\[
\min_{\alpha, \gamma, \eta, p} \sum_{s \in S} \Pr_s \sum_{t=1}^{T} \sum_{g \in G} f_g (\alpha_{gts}, \gamma_{gts}, \eta_{gts}, p_{gts})
\]

\[
- \sum_{g \in G} \sum_{b \in B} \sum_{t=t_{b1}}^{t_{b2}} \sum_{s \in S_b} \lambda_{gbts} (\alpha_{gts} - \bar{\alpha}_{gbt})
\]

s.t.

\[
(\alpha_s, \gamma_s, \eta_s, p_s) \in \mathcal{X}_s, \ \forall s \in S
\]
Retaining Nonanticipativity – The Restricted Master Problem

\[
\begin{align*}
\min_{w, \bar{\alpha}} & \quad \sum_{s \in S} P_r_s \sum_{i=1}^{n_s} c_{si} w_{si} \\
\text{s.t.} & \quad \sum_{i=1}^{n_s} A_{gtsi} w_{si} = \bar{\alpha}_{gbt}, \quad \forall g \in \mathcal{G}, b \in \mathcal{B}, s \in S_b, t = t_{b1}, \ldots, t_{b2} \\
& \quad \sum_{i=1}^{n_s} w_{si} = 1, \quad \forall s \in S \\
& \quad w_{si} \geq 0, \quad \bar{\alpha}_{gbt} \text{ free.}
\end{align*}
\]

Notation

- \(n_s\) - number of columns from subproblem \(s\)
- \(c_{si}\) - operational cost of column \(i\) for scenario \(s\)
- \(w_{si}\) - weight variable for column \(i\), scenario \(s\), \([0, 1]\)
- \(A_{gtsi}\) - commitment values of column \(i\) for scenario \(s\)

Decomposing Stochastic Unit Commitment Problems

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Retaining Nonanticipativity – The Restricted Master Problem

\[
\begin{align*}
\min_{w, \bar{\alpha}} & \quad \sum_{s \in S} Pr_s \sum_{i=1}^{n_s} c_{si} w_{si} \\
\text{s.t.} & \quad \sum_{i=1}^{n_s} A_{gtsi} w_{si} = \bar{\alpha}_{gbt}, \forall g \in G, b \in B, s \in S_b, t = t_{b1}, \ldots, t_{b2} \\
& \quad \sum_{i=1}^{n_s} w_{si} = 1, \forall s \in S \\
& \quad w_{si} \geq 0, \bar{\alpha}_{gbt} \text{ free.}
\end{align*}
\]

Issues

▶ Well-centered dual solution required for stability \(\rightarrow\) use IPMs
▶ Dual Degeneracy \(\rightarrow\) dual optimal face is commonly unbounded
▶ Noninteger solutions possible \(\rightarrow\) B&P framework required for integrality
Dual Stabilisation of the RMP

\[
\max_{\sigma, \lambda} \sum_{s \in S} \sigma_s - \epsilon \sum_{g \in G} \sum_{b \in B} \sum_{t = t_{b1}}^{t_{b2}} \sum_{t = t_{b1}}^{t_{b2}} \sum_{t = t_{b1}}^{t_{b2}} \left( \lambda_{gbts} - \hat{\lambda}_{gbts} \right)^2
\]

s.t.

\[
\sum_{g \in G} \sum_{b \in B} \sum_{t = t_{b1}}^{t_{b2}} \lambda_{gbts} A_{gtsi} + \sigma_s \leq Pr_s c_{si}, \ \forall s \in S, \ i = 1, \ldots, n_s
\]

\[
\sum_{s \in S_b} \lambda_{gbts} = 0, \ \forall g \in G, \ b \in B, \ t = t_{b1}, \ldots, t_{b2}
\]

\[
\lambda_{gbts}, \ \sigma_s \ \text{free.}
\]

Dual Notation

\(\sigma_s\) - duals of convexity constraints

\(\lambda_{gbts}\) - duals of nonanticipativity constraints
Dual Stabilisation of the RMP

\[
\max_{\sigma, \lambda} \sum_{s \in S} \sigma_s - \epsilon \sum_{g \in G} \sum_{b \in B} \sum_{t = t_{b1}}^{t_{b2}} \sum_{s \in S_b} \left( \lambda_{gbts} - \hat{\lambda}_{gbts} \right)^2
\]

s.t.

\[
\sum_{g \in G} \sum_{b \in B} \sum_{s \in S_b} \lambda_{gbts} A_{gtsi} + \sigma_s \leq Pr_s c_{si}, \forall s \in S, i = 1, \ldots, n_s
\]

\[
\sum_{s \in S_b} \lambda_{gbts} = 0, \forall g \in G, b \in B, t = t_{b1}, \ldots, t_{b2}
\]

\[
\lambda_{gbts}, \sigma_s \text{ free.}
\]

Dual Notation

- \(\sigma_s\) - duals of convexity constraints
- \(\lambda_{gbts}\) - duals of nonanticipativity constraints
Further Relevant Issues

- Warmstart: initial columns and proximity term estimate
- Linear approximation of convex quadratic or PWL costs in the RMP
- Ramp rate constraints → include nonanticipativity on power outputs
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Summary: The Column Generation Algorithm

Flowchart of the column generation method.

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## Numerical Tests

- AMPL script (serial), uses parallel CPLEX to solve LPs and MIQPs
- FERC RTO TS, 1011 Generators, 24h ahead, 2-10 Scenarios, 0.05% Gap
- Linux 64bit 128GB RAM, Dual 8 Core Intel Xeon
- Observation: RMP solution integer, or integer solution among columns within Gap
Numerical Tests – Required Iterations

Iterations required to converge to a 0.05% RMP duality gap plotted against the scenario count. We use $\epsilon = 10^{-4}$ (left) and $\epsilon = 10^{-3}$ (right).
Convergence of upper and lower RMP bounds with a variable metric (left), $\epsilon = 10^{-4}$ (center) and $\epsilon = 10^{-3}$ (right). This is the eight scenario example.
Conclusion

- Solving Stochastic Unit Commitment directly by B&B is not practical.
- Various decomposition schemes can be used to make the problem more tractable. Scenario decomposition is the most promising one.
- Branch & Price can guarantee optimality, unlike e.g. Progressive Hedging.
- Numerical results suggest that branching will hardly be necessary.
- The approach scales well as the number of scenarios increases.
- Primal and dual initialization is key for robustness.
Thank you for your attention.

To get the report:
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Stochastic scenario decomposition for multi-stage stochastic programs.

RTO unit commitment test system.

M. E. Lübbecke and J. Desrosiers.
Selected topics in column generation.
G. Lulli and S. Sen.
A branch-and-price algorithm for multistage stochastic integer programming with application to stochastic batch-sizing problems.

T. Shiina and J. R. Birge.
Stochastic unit commitment problem.

S. Takriti, J. R. Birge, and E. Long.
A stochastic model for the unit commitment problem.

F. Vanderbeck.
Implementing mixed integer column generation.
J. Wilkes, J. Moccia, and M. Dragan.