Thoughts Originating in the Talks

Suggested by the Lectures We Attended,

the Discussions We Had,

the Questions We Asked and

the Answers I Failed to Understand

Epifanio G. Virga
Department of Mathematics
University of Pavia, Italy

Summary

Surfaces
Defects
Activity
Phases

Surfaces

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- Would a dependence on $\nabla_{\mathrm{s}}\mathbf{Q}$ (elaborating on the original idea of Berreman) be appropriate to model surface inhomegeneities and how would it affect the analysis of the total energy minimizers? (C. Zannoni)

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- In the ***high*** concentration limit, could the resultant system be treated in De Giorgi's Γ-limit approach?

  (P. Bauman, C. Calderer)

- For dipolar colloids (e.g., endowed with a permanent magnetic dipole) what a rigorous estimate of the DE GENNES-PINCUS ***aggregation parameter*** would tell about preventing chaining and cluster formation? (T. SLUCKIN)

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- ***Decorated*** vesicles, described by deformable surfaces endowed with some kind of ***order*** may acquire non regular equilibrium shapes, some ***faceted*** in the ***floppy*** limit. Could a Γ-limit approach construct an effective ***line tension*** that arises only wherever the unit normal is discontinuous, in a way reminiscent of the SBV space? (M. BOWICK)

Defects

- New ***metastable*** defect structures have emerged, such as the ***torons*** seen in chiral nematics. No analytic study exists for such structures. Could the topological methods on non-linear analysis devised to count and characterize the critical points of a functional be useful here? (I. Smalyukh)

Defects

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- Colloidal nanoparticles seem to have an affinity for defects. When dispersed in a liquid crystal, they are attracted towards the defect cores. Could it be that they somehow feel the surrounding "metric"? Would geometric tools describe their motion more appropriately?
(G. Gibbons, R. Kamien, S. Kralj)

- Colloids induce defects which could help propelling them with the intervention of an external field, in a ***semi-active*** fashion. For example, both a hyperbolic ***point*** defect and a ***loop*** defect could be induced by a spherical colloid. No ***exact*** analytical study is known which characterizes the transition from one to the other. How would an external field affect such a transition?

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- In nematic ***elastomer*** thin films one can imprint defect patterns which can be acted upon by both light and heat. Has an effectively two-dimensional non-linear elastic theory for these decorated films been developed? How in such a theory would defects be driven by the curvature field of the deformed film?

  (M. Warner)

- How does light interact with nematic order? And how does it transfer momentum to the fluid? These questions are related to the century-long Abraham-Minkowski controversy about the correct expression for the light momentum in a medium.

  (M. Čopič, S. Barnett)

Activity

- A continuum theory for active nematics should be suggested by (or at least be in agreement with) the properties of the flow induced by microscopic ***swimmers*** in a viscous fluid. For ***dipolar*** swimmers, the accepted active stress is

$$\mathbf{T}_{\mathrm{a}} = S\mathbf{Q}, \qquad \mathbf{Q} = \left\langle p^2 \boldsymbol{e} \otimes \boldsymbol{e} - \frac{1}{3}\mathbf{I} \right\rangle.$$

But, what if swimmers are ***quadrupolar***? How would the linear stability analysis of active flow known for the dipolar case be affected? (C. Marchetti, T. Pendley, J. Yeomans)

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But, what if swimmers are ***quadrupolar***? How would the linear stability analysis of active flow known for the dipolar case be affected? (C. Marchetti, T. Pendley, J. Yeomans)

- What kind of ***averages*** are involved in deriving the macroscopic active stress from the microscopic force distribution on swimmers? (T. Liverpool)

- One can get around PURCELL's scallop theorem by considering ***flexible*** bodies. How is the hydrodynamic efficiency of flexible filaments actively oscillating in a viscous fluid affected by the presence of hydrodynamic slip?

Phases

- Density functional theory is a variational formulation for the statistical mechanics of classical particles in which the function to be determined is the probability density itself in phase-space. Once formulated in ***canonical*** ensemble, is this theory capable of describing multi-species distribution or even polydispersity?

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- And to predict phase mixing and phase de-mixing of particles interacting purely in an entropic (excluded-volume) fashion?
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- In the the mean-field theory of liquid crystals, phases may also appear as solutions to a functional fixed-point problem. Can the topological theory of fixed-points be useful in classifying all possible phases?

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