Liquid crystal defects, a critical overview

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Defects in liquid crystals present an extremely vast set of situations, depending on the nature of the order parameter, elasticity properties, boundary conditions.

One distinguishes : defects characteristic of liquid crystalline media with one or two discrete repeat distances (smectic and columnar phases) which can be investigated by pure geometric methods ; defects characteristic of a director order parameter, which employ the Volterra process (for one dimensional defects) and topological methods, those latter providing scope for defects of various dimensionalities ; finally continuous defects whose consideration is essential for a description of defect physical properties.

These topics will be confronted with an historical point of view and possible future developments.

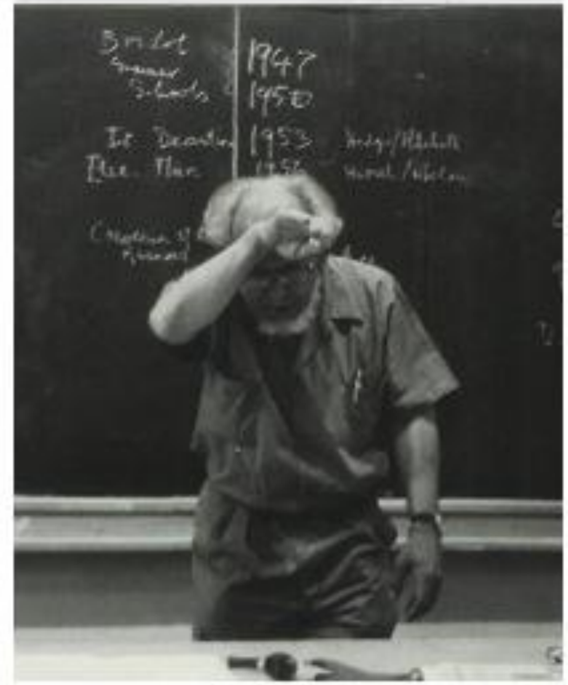
Cambridge UK 24 June 2013

Georges Friedel (1867-1933)

Crystallographer, accessorily geologist and mineralogist.
MESOMORPHIC PHASES.
classification
Pioneering investigations on DEFECTS in condensed matter.

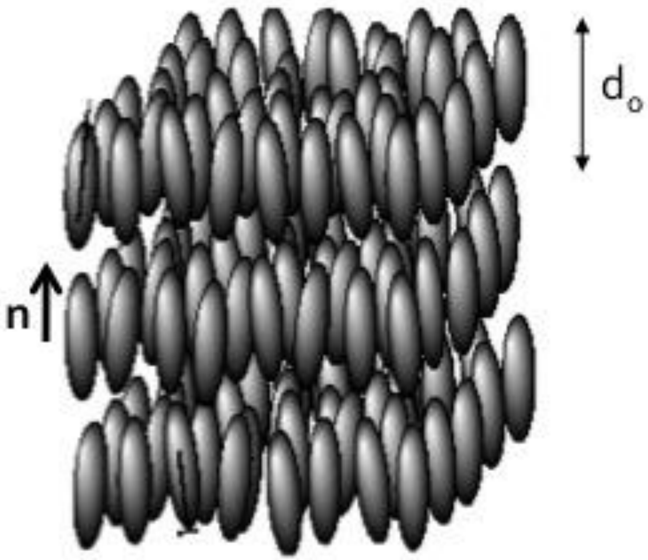
Les états mésomorphes de la mat!ère
Annales de Physique18 (1922) 273-474

Sir Charles Frank (1911-1998)

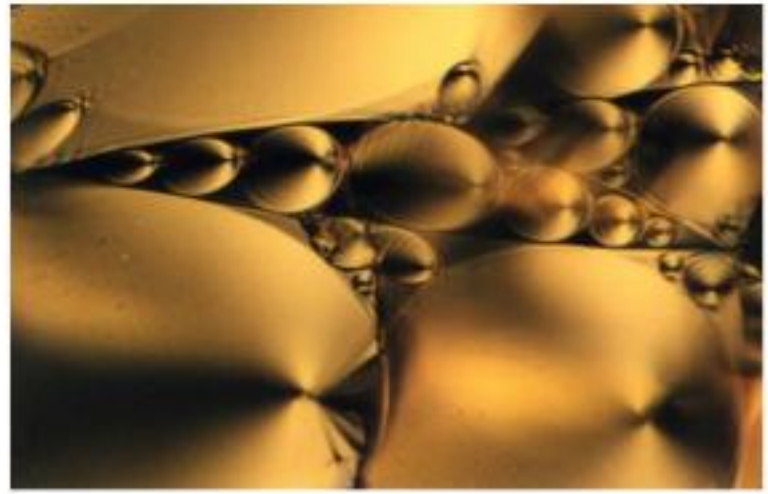


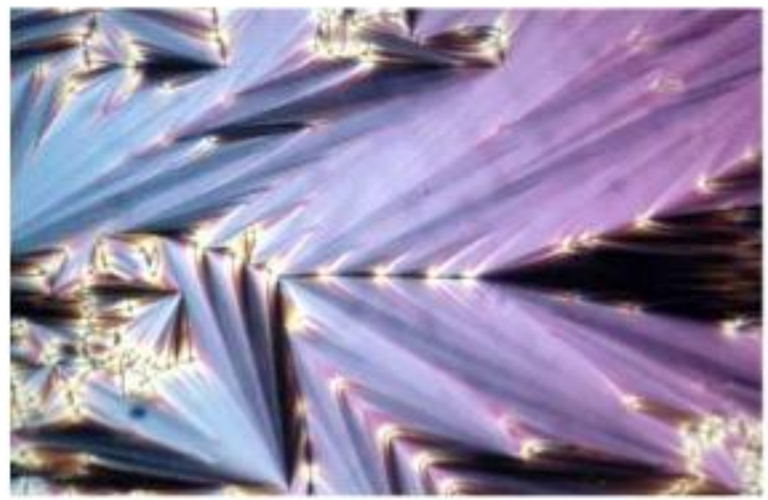
Crystal plasticity: DISLOCATIONS.
Liquid Crystals: DISCLINATIONS, ELASTICITY.
Local order ICOSAHEDRIC in supercooled liquids and some intermetallic alloys
(Frank & Kasper phases).
Polymers, cold fusion, geophysics.

Liquid Crystals
Disc Far Soc 25 (1958) 1

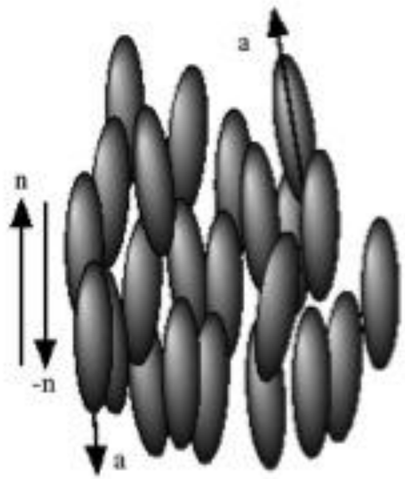


lamellar phase (SmA)
the layers are fluid



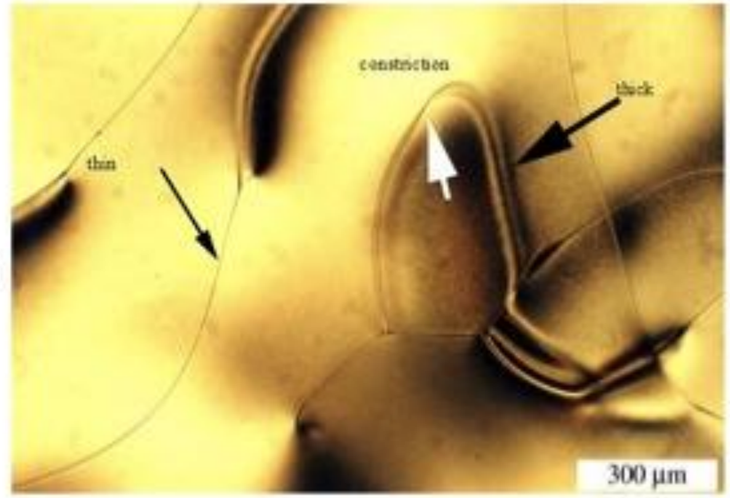


focal conic domains

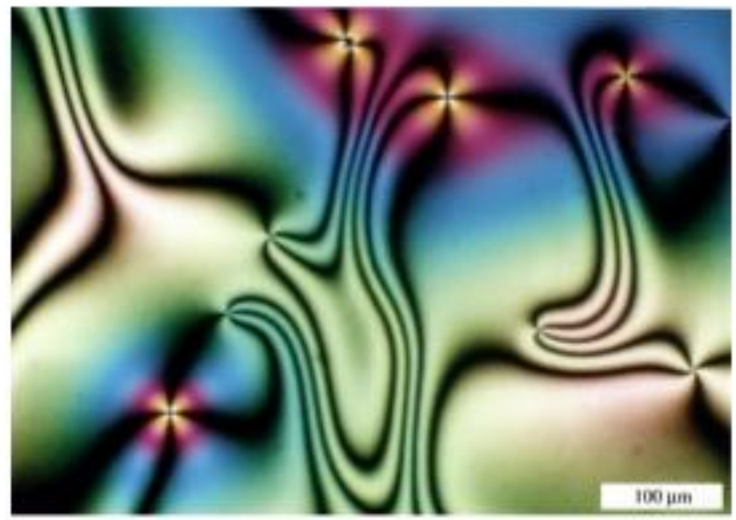


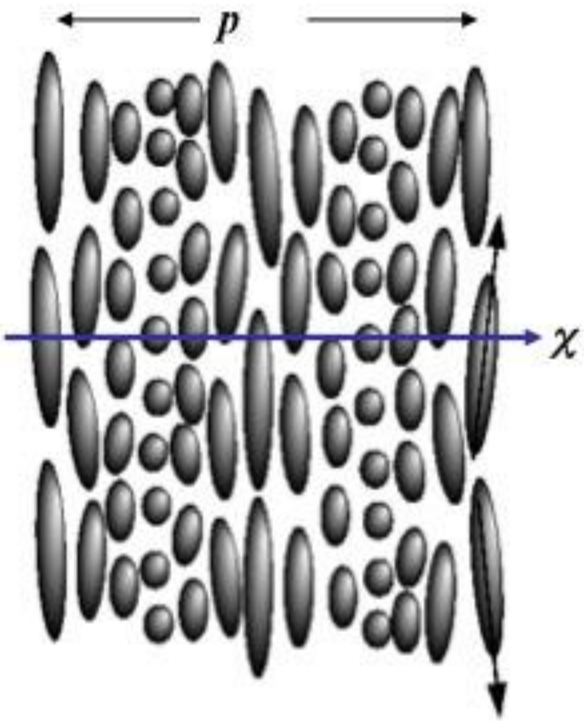
nematic phase
the molecules are
statistically parallel
to the director

thins and thicks



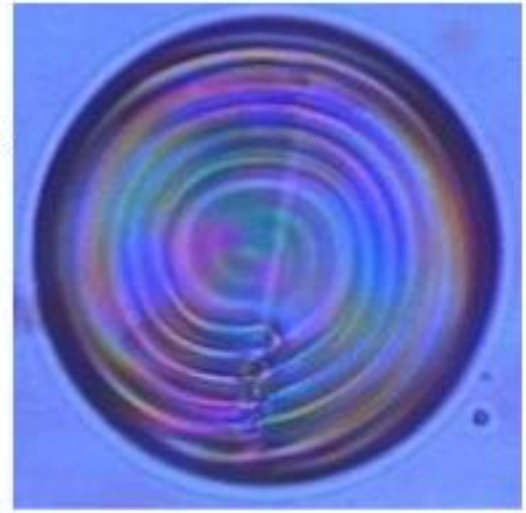
>o and <o nuclei





twisted nematic (cholesteric);
the axis of twist χ is
perpendicular to the molecules

double helical disclination
cholesteryl myristate
droplet in glycerol
(Robinson spherolite)



dislocations
(fingers)

isometric textures

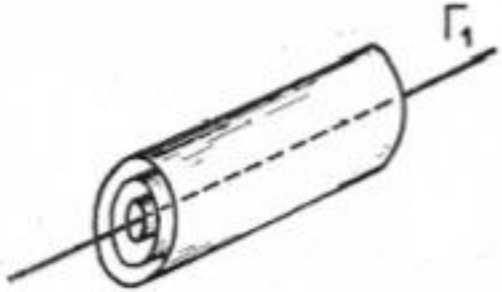
focal conic domains

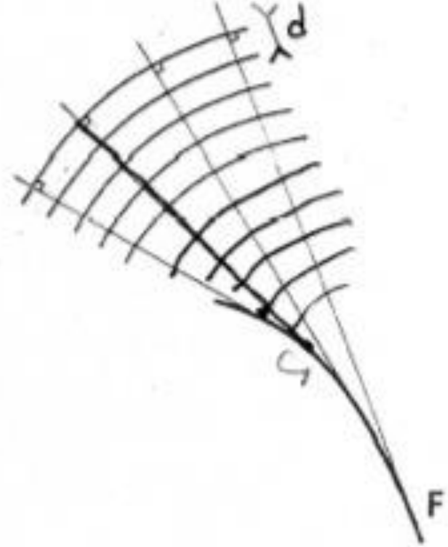
devopable domains

flexible defect lines (dislocations, disclinations) and other dimensionalities

the topological theory of defects

developments


Γ_1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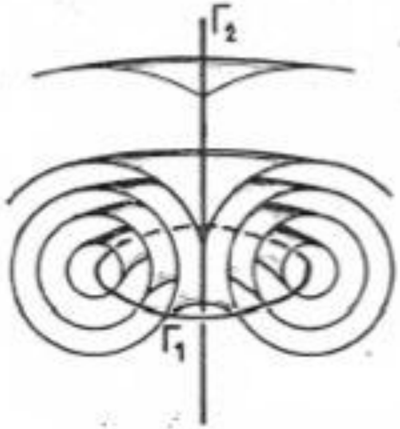
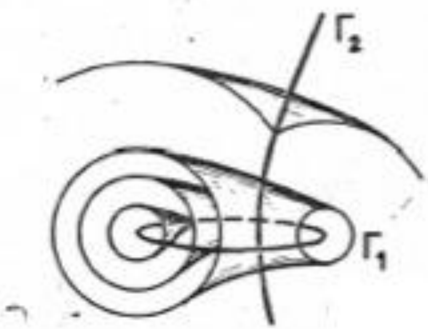
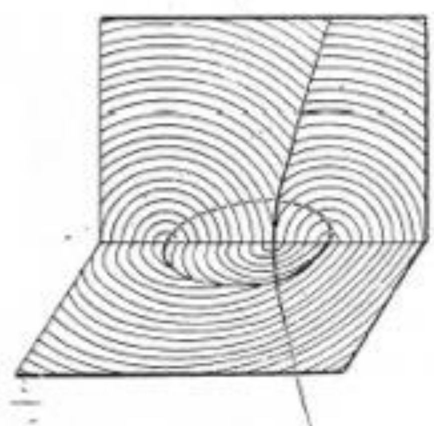
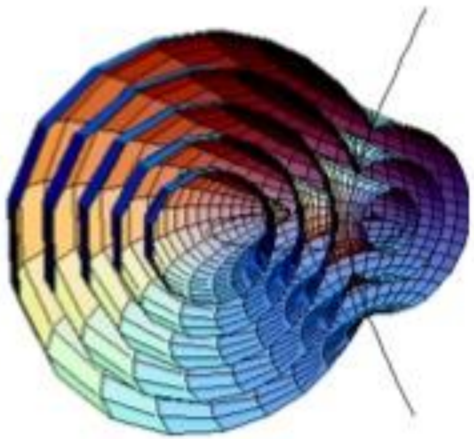
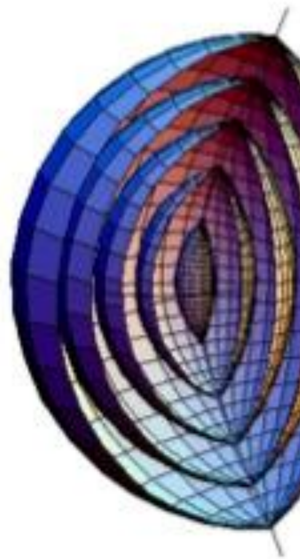

Γ_2
Γ_1

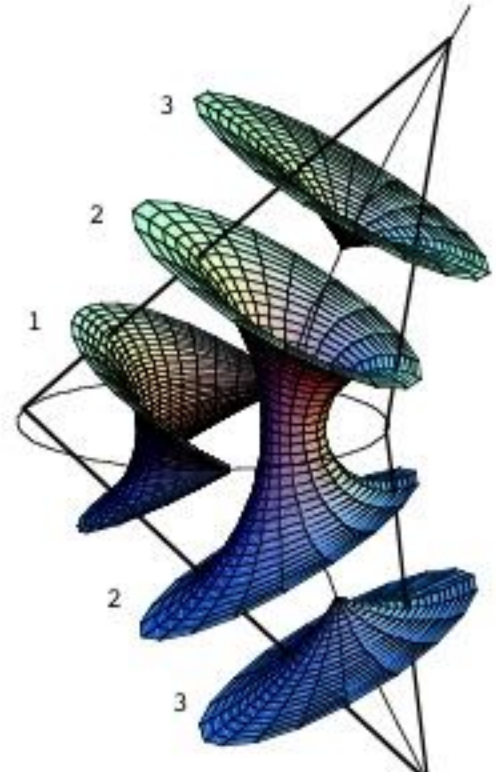

Γ_2
Γ_1







$\mathcal{K} = \sigma_1\sigma_2 > 0$

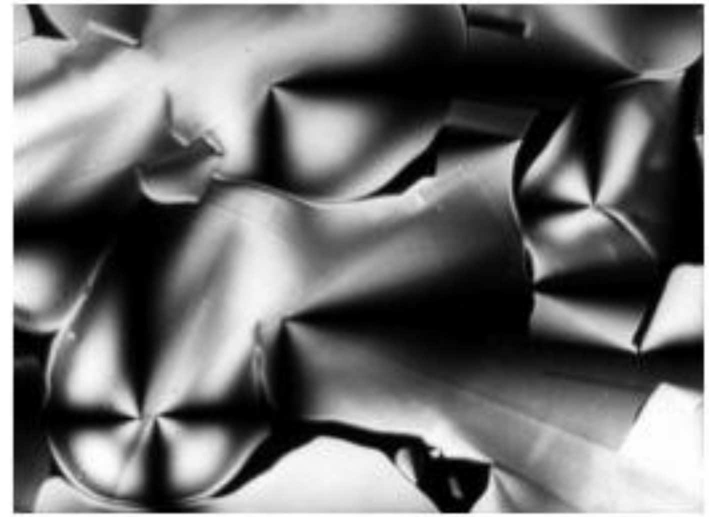


$\mathcal{K} = \sigma_1\sigma_2 < 0$

Boltenhagen, Lavrentovich, MK

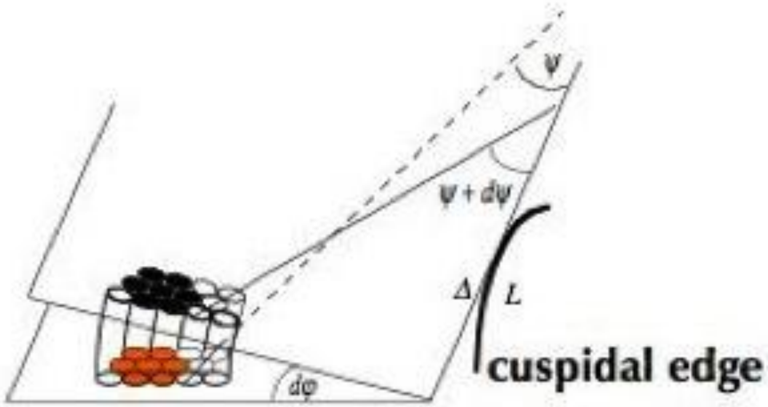
columnar phase
the columns are
liquid like

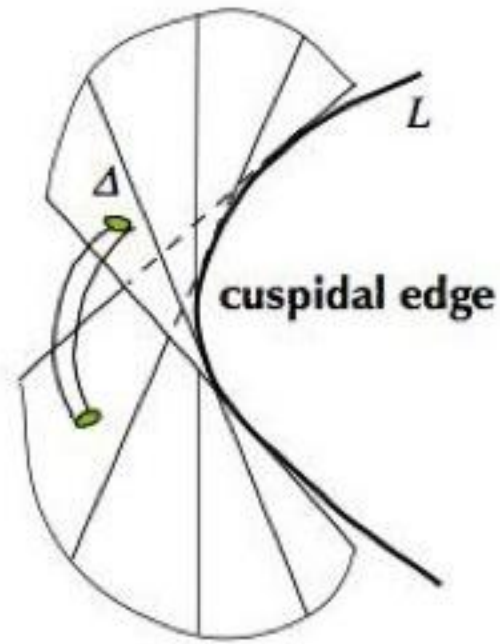
Oswald, Kleman 1982



developable domain degenerate into a cylinder
the cylinder is viewed end-on, (left bottom corner)
and is split into two half cylinders

geometry of the columns around the cylinder

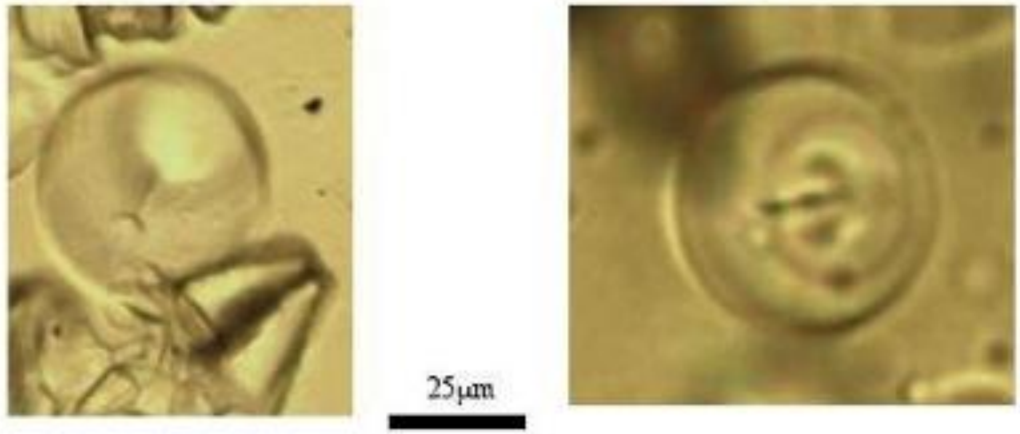




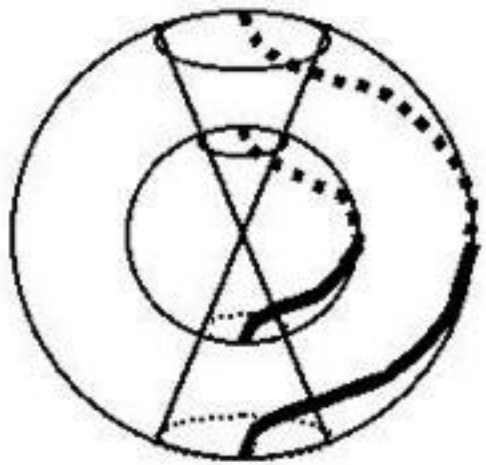
the planes orthogonal to the columns remain
In the distorted columnar phase. They become
the osculating planes of a curved line L.

the developable surface D is the
envelop of these planes

MK 1982



the developable is a cone



Nastyshyn 2002

isometric textures

focal conic domains

devopable domains

flexible defect lines and other dimensionalities

the topological theory of defects

developments

order parameter space V:

space of all the possible different orientations and positions of the ordered substance in its ground state, group G (=E(3));
H, symmetry group of the substance (little group),

⟶ quotient space V=G/H

defect L, circuit C, dim L = d, dim C = 2-d

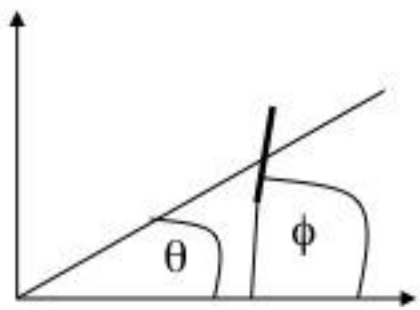
point defect, dim C = 2; line defect, dim C = 1; wall, dim C = 0
each position and orientation along C is 1-1 with a left coset of H.

homotopy classes in V of the circuits C'

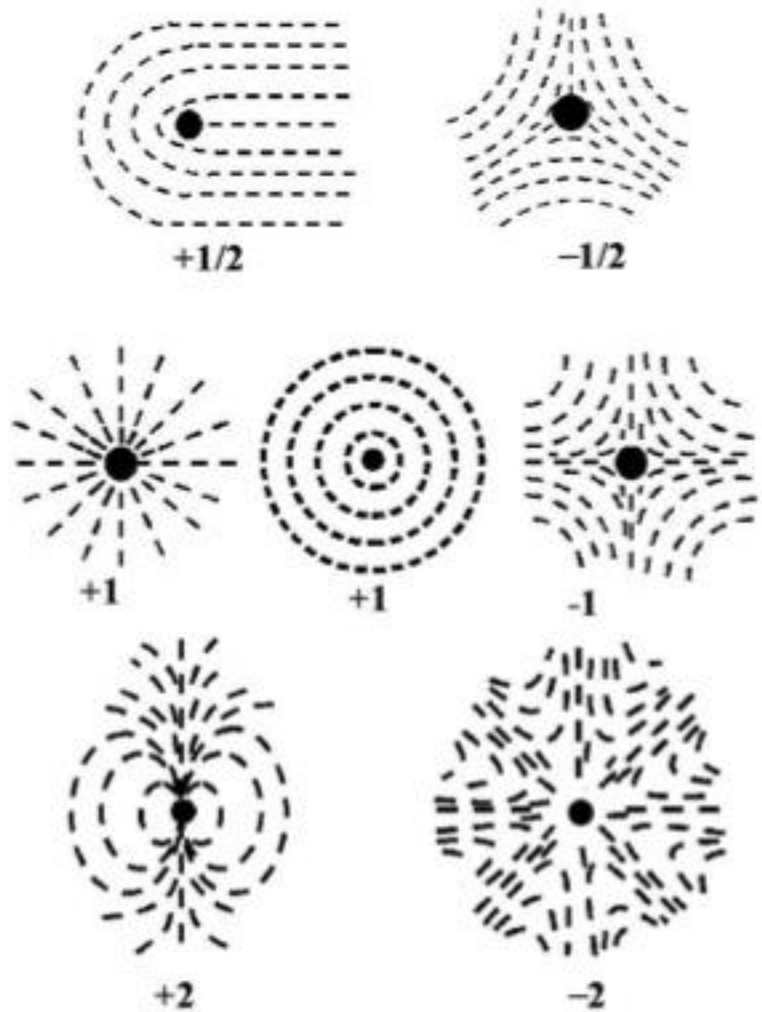
C' = image of C in V

$\Pi_2(V)$ $\Pi_1(V)$ $\Pi_0(V)$

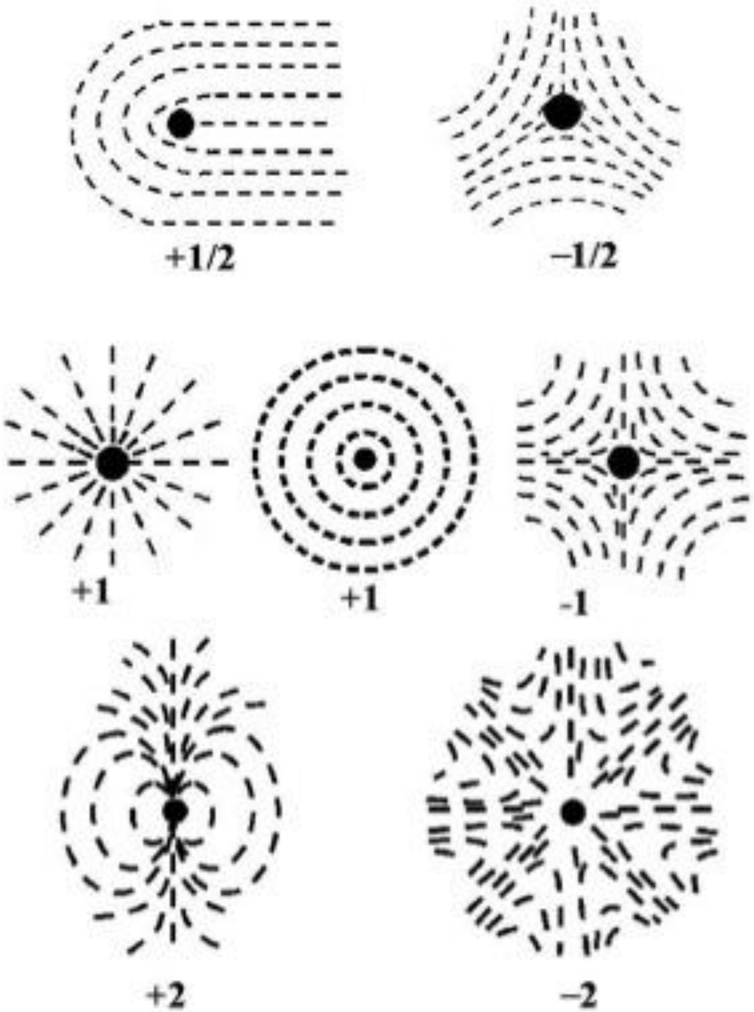
reviews: Mermin, Michel, Trebin

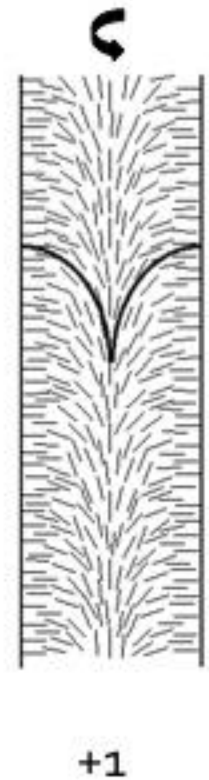


$$\phi = (n/2)\theta + \phi_0$$

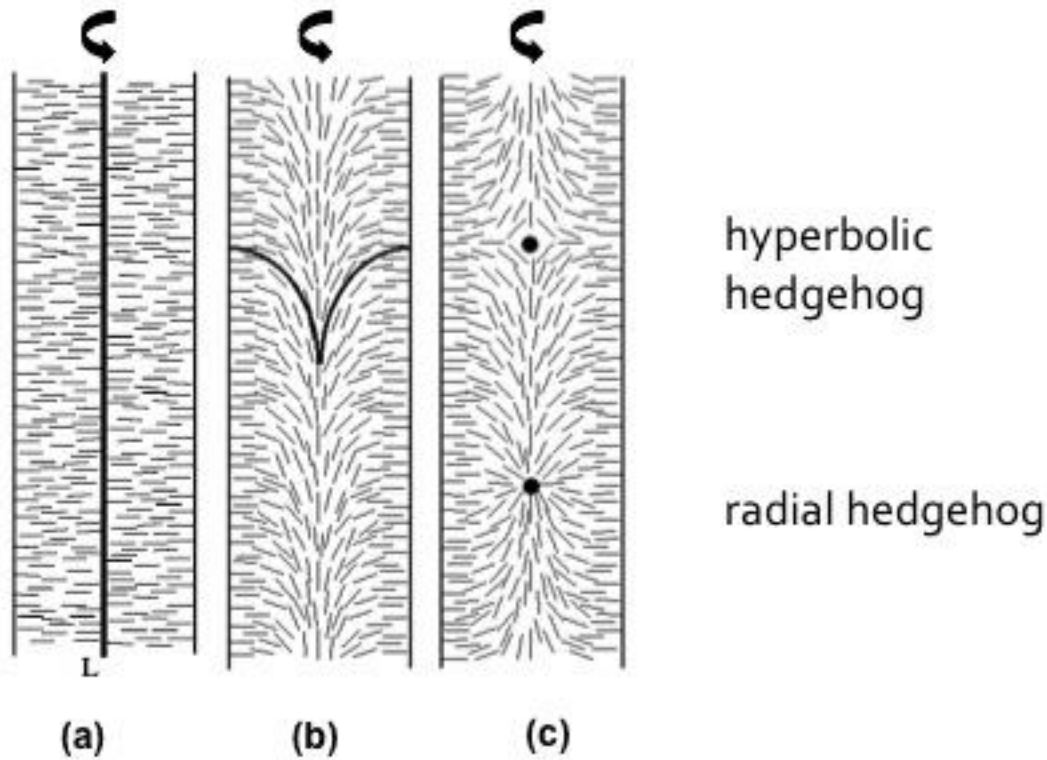


Oseen, Frank,



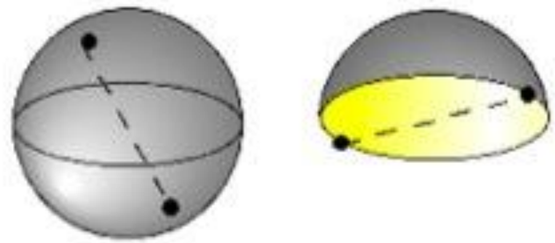


deGennes, R. Meyer, P. Cladis, MK,

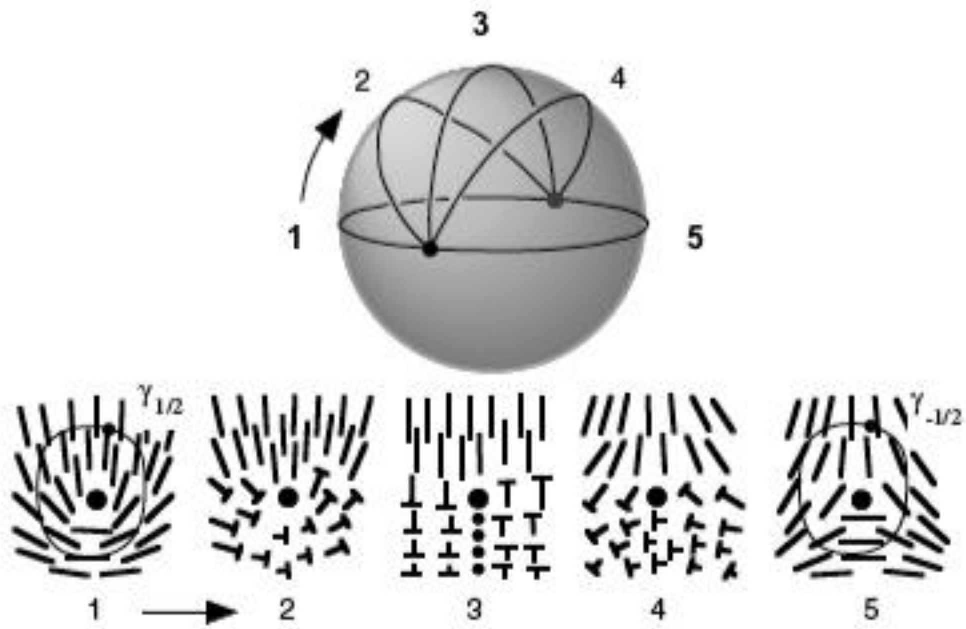


meridian cuts, cylindrical sample: (a)- singular core, (b)- escaped core along the third dimension, (c)- singular points

P. Cladis, M. Williams



projective plane $P_2 = So(3)/D_\infty$ $\Pi_1(P_2) = Z^2$



topological equivalences

Finkelstein, G. Toulouse and MK., 1978, Rogula, MK and L. Michel, R. Thom

Bouligand, Y., 1981, in Physics of Defects, Les Houches Session XXXV, North-Holland.

topological theory of defects

properties

-non-commutativity of the fundamental group

- configurations: $\Pi_3(V)$

beyond

-* topological interaction between point and line defects (nematics)

-* knots and links of line defects (nematics)

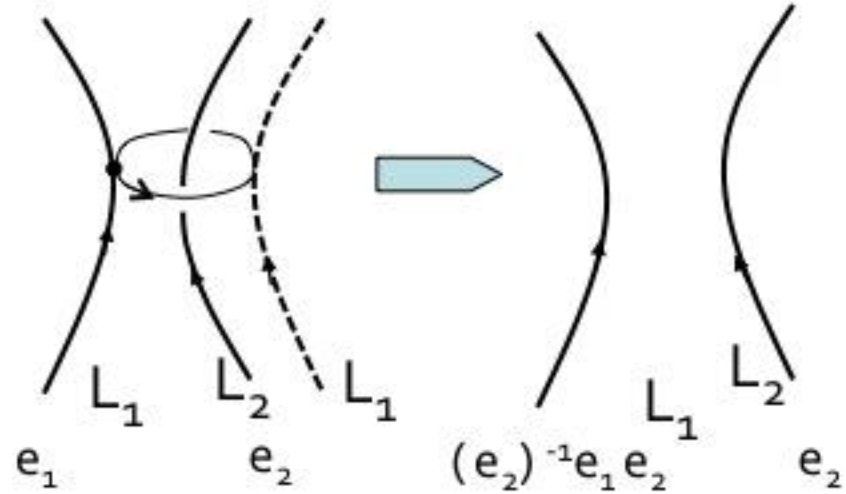
-* integrability conditions

- frustration

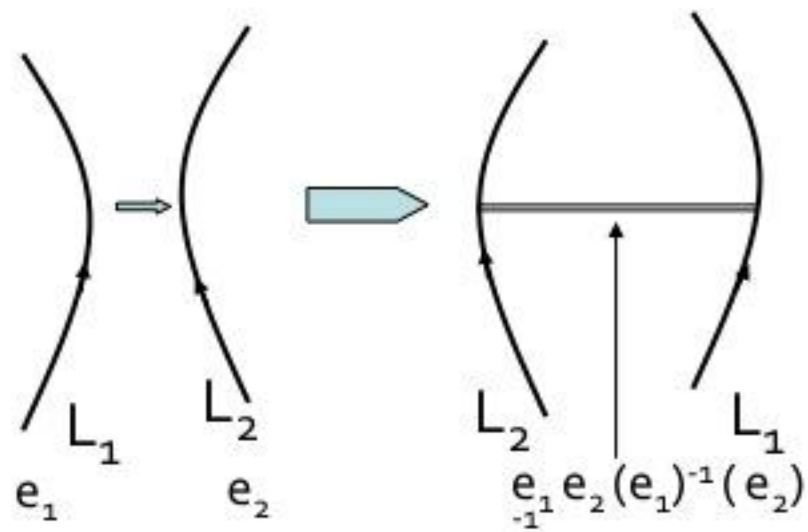
-interplay between dislocations and disclinations

non-commutativity

biaxial nematics (G. Toulouse)
cholesterics (Volovik, Mineev)
He3
$\Pi_1 = Q$ (group of quaternions, 8 elements, 5 classes of conjugacy)



Poenaru, Toulouse

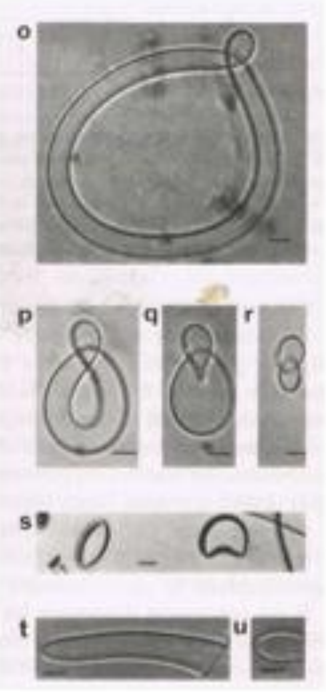


configurations

$\Pi_1(S_1)$ Finkelstein, Misner Sine-Gordon equation

$\Pi_2(S_2)$ Belavin, Poliakov nematic capillary

$\Pi_3(S_2)$ Bouligand et al. 1978 cholesteric
The Hopf map



Bouligand 1974

topological theory of defects

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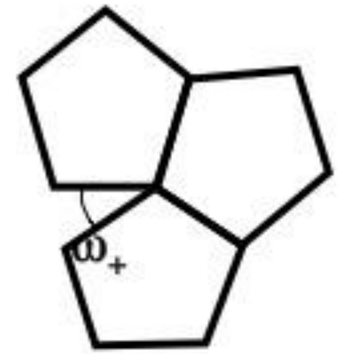
geometrical frustration Anderson, Toulouse spin-glasses

MK, Sadoc 1979 metallic glasses

in 2D, S_2 tiled with regular pentagons (dodecahedron),
flattened on E(2) with disclinations
in 3D, S_3 tiled with regular icosahedra,
flattened on E(3) with disclinations

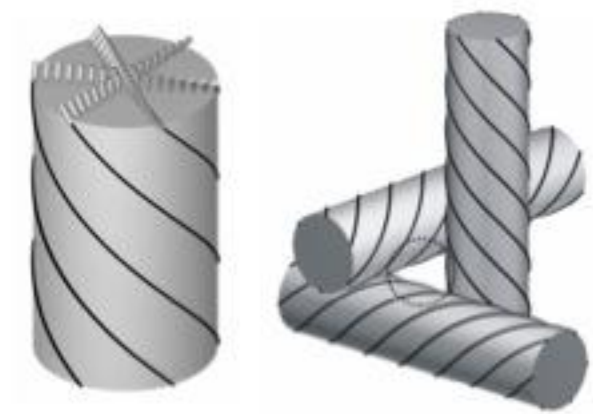


Frank and Kasper phases 1958

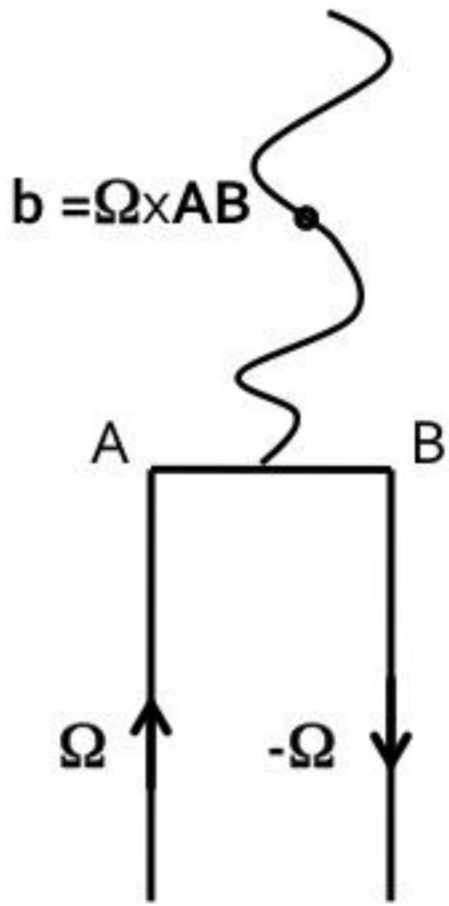
Sethna et al. 1981, Pansu et al. 1986
blue phases
local order: double-twisted cylinders, frustrated

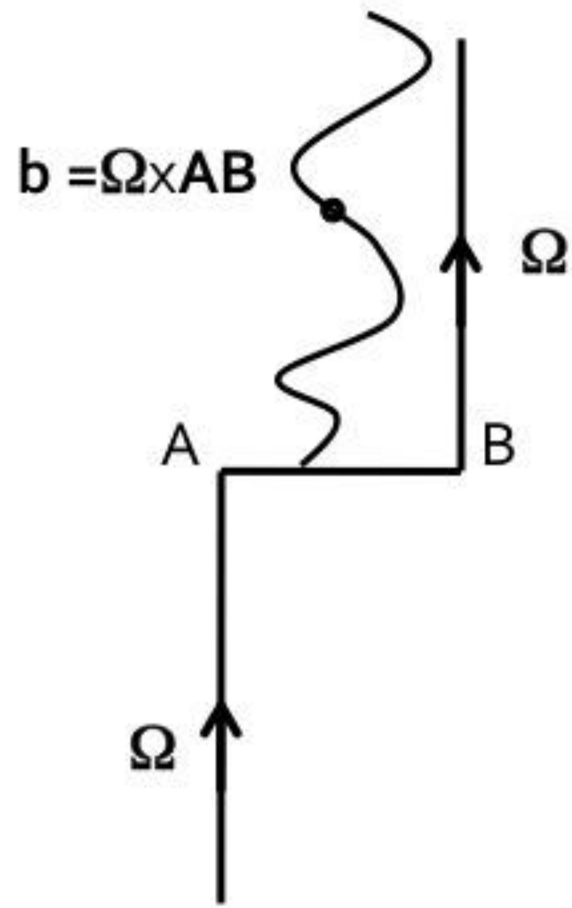
regular assembly of cylinders, separated by disclinations

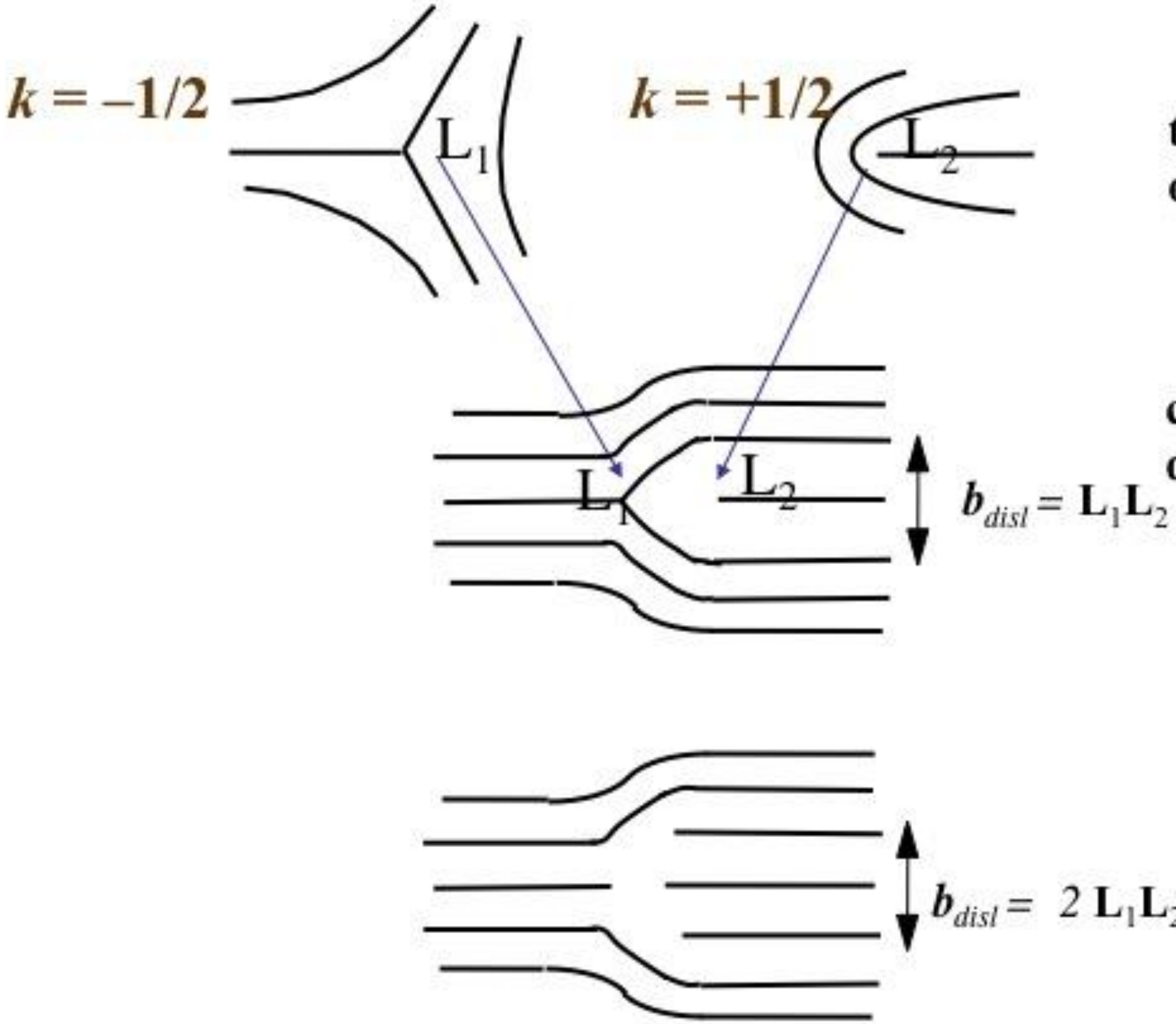
curved space representation in S_3:
Clifford parallels, Hopf fibration



interplay dislocations-disclinations



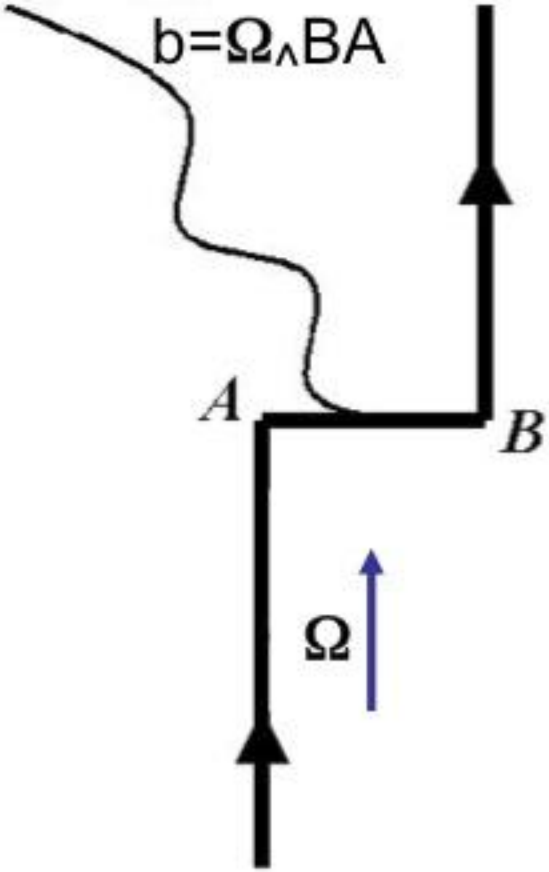




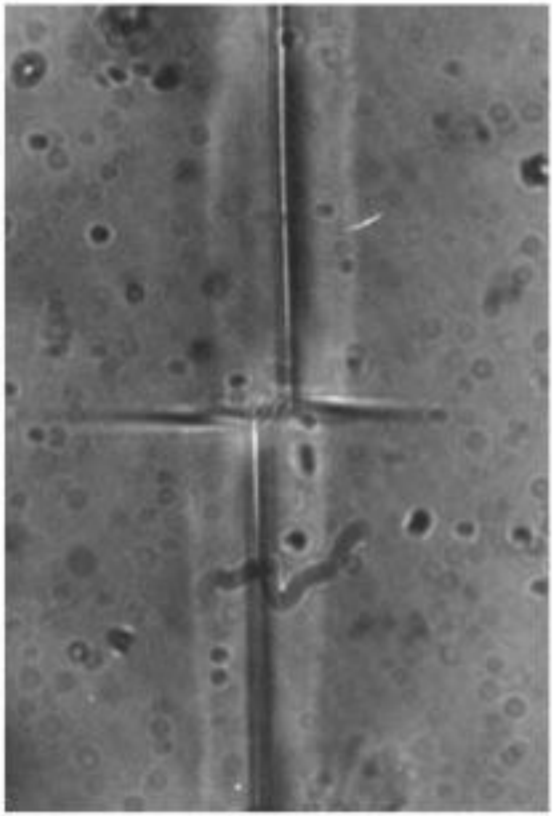
two disclinations of opposite signs

coupled at short distance L_1L_2

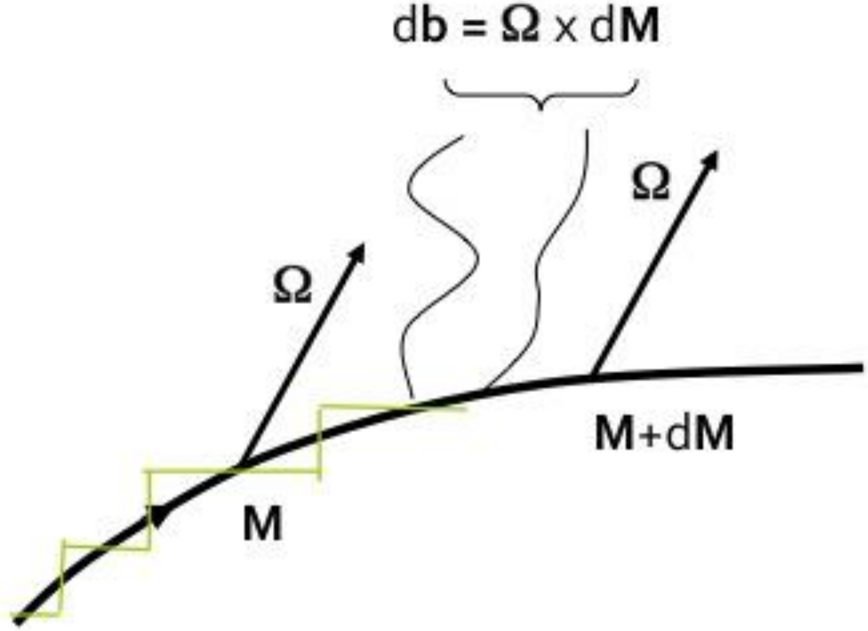
equivalence to dislocations



kink: theoretical model



kink: disclination in a columnar phase (Patrick Oswald)



MK and J. Friedel RMP 2008

Conclusion

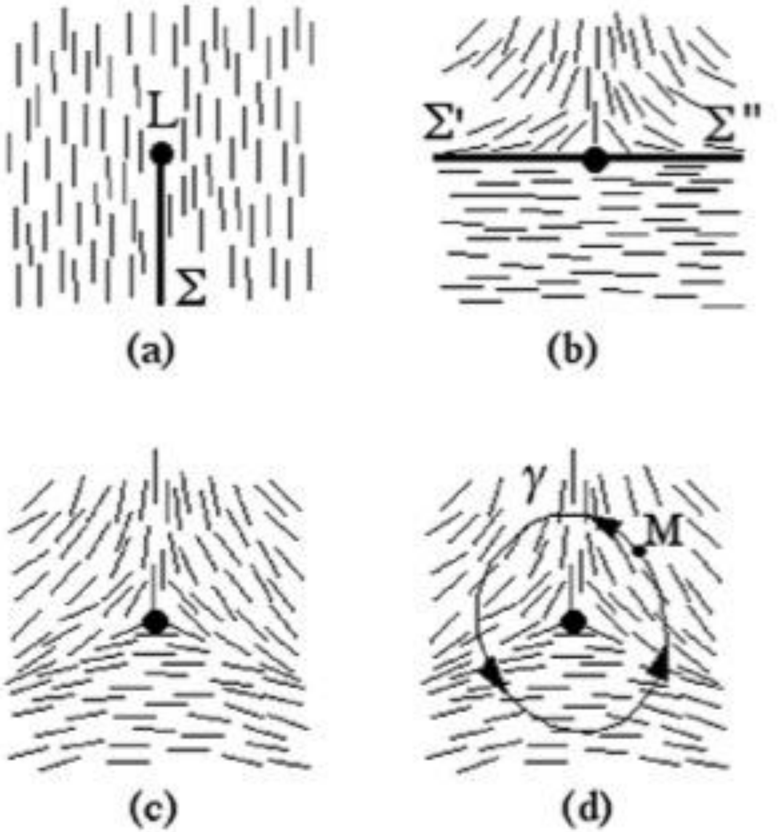
Important successes in mathematics, algebraic topology and differential geometry .

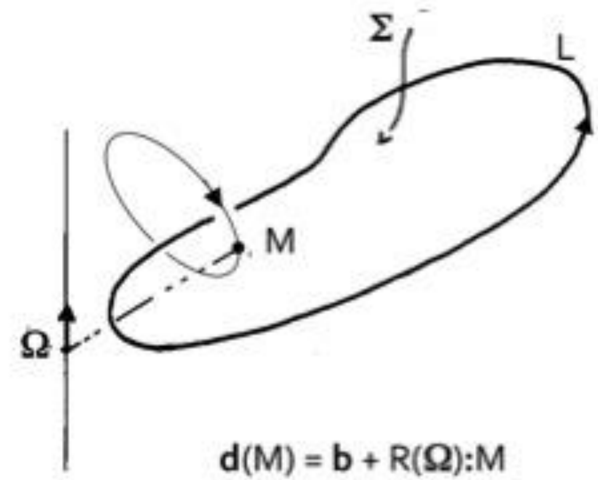
Geometry is revisited at a fast rate: nodes and links, braids, quaternions, Hpf mappings. some of these topics were hot 20 years ago, and their revival stresses their importance as basic concepts.

About experimental observations: progresses in the methods of direct observations, confocal microscopy, fluorescence, Laguerre-Gauss vortex beams, etc, supplemented for their interpretations with modern computing techniques.

Some future developments?:

- cores of defects, theory and experiments,
- rheological properties of defects, cf. crystal plasticity; the Volterra point of view
- dispirations


L
Σ
(a)
Σ'
Σ"
(b)
(c)
γ
M
(d)


Σ
L
M
Ω
d(M) = b + R(Ω):M