A Top-Down Model for Lifshitz Holography

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Holography: from gravity to quantum matter

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Outline of the Talk

- Motivation
- The Model and Lifshitz Solutions
- AlLif Space-Times and Beyond
- Asymptotics and Boundary Geometry
- Newton-Cartan
- Ward Identities and the boundary stress tensor
- Counting Sources and Vevs
- Summary & Outlook

Motivation

- Lifshitz holography [Kachru,Liu,Mulligan, 2008],[Taylor, 2008]: New type of holographic duality with non-AdS asymptotics and Lorentz violating dual field theories. Here z=2: $[t] = 2\,, [\vec{x}] = 1.$

- Interesting for: i). AdS/CMT as an IR geometry which more generally can be hyperscaling violating, ii). extending holography beyond AdS/CFT.

- Boundary geometry and boundary stress-energy tensor for Lifshitz holography.

- Deformations of AlLif space-times.

- Identify the sources & vevs: holographic reconstruction.

The Model

- Truncation of type IIB on S^5

$$S = \int d^5x\sqrt{-\hat{g}}\left(\hat{R} + 12 - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2}e^{2\hat{\phi}}(\partial\hat{\chi})^2\right)$$

- Scherk-Schwarz reduce to 4D using standard KK ansatz for metric and dilaton and $\hat{\chi} = ku + \chi$

$$S = \int d^4x\sqrt{-g}\left(R - \frac{3}{2}(\partial\Phi)^2 - \frac{1}{4}e^{3\Phi}F^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(D\chi)^2 - V\right)$$

$$D\chi = d\chi - kA \qquad F = dA \qquad V = \frac{k^2}{2}e^{-3\Phi+2\phi} - 12e^{-\Phi}$$

- [Donos, Gauntlett, 2010],[Cassani, Faedo, 2011],[Chemissany, J.H., 2011]

- Counterterms and reduction: [Papadimitriou, 2011],[Chemissany, Geissbuehler, J.H., Rollier, 2012]

Lifshitz Solutions

- A z=2 Lifshitz background:

$$ds^2 = l^2 e^{\Phi(0)} \left(\frac{dr^2}{r^2} - e^{-2\Phi(0)} l^2 \frac{dt^2}{r^4} + \frac{1}{r^2}(dx^2 + dy^2) \right) \qquad e^{\Phi} = e^{\Phi(0)} = \frac{lkg_s}{2}$$

- In 5D: $d\hat{s}^2 = l^2 \left(\frac{dr^2}{r^2} + \frac{1}{r^2}(2dudt + dx^2 + dy^2) \right) + \frac{l^2 k^2 g_s^2}{4} du^2$

- Size of the circle: $2\pi L_{\text{phys}} = \int_0^{2\pi L} du \sqrt{\hat{g}_{uu}} = 2\pi L \frac{lkg_s}{2}\,, \qquad u \sim u + 2\pi L$

- Validity of the approximation: $\frac{l}{l_s} \gg 1\,, \;\; g_s \ll 1\,, \;\; \frac{L_{\text{phys}}}{l_s} \gg 1$

- $\frac{L_{\text{phys}}}{l} = \frac{Lkg_s}{2} \gg 1$ (decompactification: $\mathcal{N} = 4$ SYM)

  ~ 1 (DLCQ with nonzero theta angle)

  $\ll 1$ (below KK mass scale: 3D LCS)

- [Costa, Taylor, 2010], [McGreevy, Balasubramanian, 2011]

AlLif Solutions and Beyond

- Asymptotically locally Lifshitz (AlLif) spaces [Ross, 2011]:

$$ds^2 = e^{\Phi}\frac{dr^2}{r^2} - e^{\underline{t}}e^{\underline{t}} + \delta_{\underline{i}\underline{j}}e^{\underline{i}}e^{\underline{j}} \qquad e^{\underline{t}}_a = r^{-2}\tau_{(0)a} + \dots \text{, where } \tau_{(0)a} \text{ is HSO}$$
$$e^{\underline{i}}_a = r^{-1}e^{\underline{i}}_{(0)a} + \dots$$

- Φ and ϕ are asymptotically constant with $\Phi_{(0)} = \log\frac{kg_s}{2}$.

- Boundary gauge field and axion:

$$A_a - e^{-3\Phi/2}e^{\underline{t}}_a = A_{(0)a} + \dots$$
$$\chi = \chi_{(0)} + \dots$$

- $\partial_a\tau_{(0)b} - \partial_b\tau_{(0)a} = 0$ there exists ADM slicing with $[t] = 2\,, [\vec{x}] = 1$

- Deformations: $\phi_{(0)} \neq \text{cst}$ and $\tau_{(0)a}$ is not HSO.

- Cannot lift the constraint: $e^{2\Phi_{(0)}} = -\frac{1}{4}e^{3\Phi_{(0)}}\left(\epsilon^{abc}_{(0)}\tau_{(0)a}\partial_b\tau_{(0)c}\right)^2 + \frac{k^2}{4}e^{2\phi_{(0)}}$

Boundary Geometry I

- We work in the gauge (5D radial gauge):

$$ds^2 = e^{\Phi}\frac{dr^2}{r^2} - e^{\underline{t}}e^{\underline{t}} + \delta_{\underline{i}\underline{j}}e^{\underline{i}}e^{\underline{j}}$$

- with Lif UV boundary conditions:

$$e^{\underline{t}}_a = r^{-2}\tau_{(0)a} + \ldots \qquad e^{\underline{i}}_a = r^{-1}e^{\underline{i}}_{(0)a} + \ldots$$

$$e^a_{\underline{t}} = -r^2\tau^a_{(0)} + \ldots \qquad e^a_{\underline{i}} = re^a_{(0)\underline{i}} + \ldots$$

$$\tau_{(0)a}\tau^a_{(0)} = -1 \qquad \tau_{(0)a}e^a_{(0)\underline{i}} = 0$$

$$e^{\underline{i}}_{(0)a}\tau^a_{(0)} = 0 \qquad e^{\underline{i}}_{(0)a}e^a_{(0)\underline{j}} = \delta^{\underline{i}}_{\underline{j}}$$

- To find out what kind of metric structure we can build from these boundary frame fields we look at how they transform under bulk local Lorentz transformations.

Boundary Geometry II

- Bulk local Lorentz transformations act on the boundary frame fields as Galilean boosts and rotations:

$$\delta\tau_{(0)a} = 0 \qquad \delta\tau^{a}_{(0)} = \xi^{b}_{(0)}\omega^{\underline{i}}_{(0)b}e^{a}_{(0)\underline{i}}$$

$$\delta e^{\underline{i}}_{(0)a} = \xi^{b}_{(0)}\left(\omega^{\underline{i}}_{(0)b}\tau_{(0)a} + \omega_{(0)b}{}^{\underline{i}}{}_{\underline{j}}e^{\underline{j}}_{(0)a}\right) \qquad \delta e^{a}_{(0)\underline{i}} = -\xi^{b}_{(0)}\omega_{(0)b}{}^{\underline{j}}{}_{\underline{i}}e^{a}_{(0)\underline{j}}$$

- Light cones flatten out. There is no Lorentzian boundary metric! Cannot raise/lower indices.

- Can only define degenerate metrics like $\Pi^{ab}_{(0)} = \delta^{\underline{ij}}e^{a}_{(0)\underline{i}}e^{b}_{(0)\underline{j}}$ and $\tau_{(0)a}\tau_{(0)b}$

- Spin connection coefficients determined by the vielbein postulate:

$$\mathcal{D}_{(0)a}e^{\underline{i}}_{(0)b} = \partial_a e^{\underline{i}}_{(0)b} - \Gamma^{c}_{(0)ab}e^{\underline{i}}_{(0)c} + \omega^{\underline{i}}_{(0)a}\tau_{(0)b} + \omega_{(0)a}{}^{\underline{i}}{}_{\underline{j}}e^{\underline{j}}_{(0)b} = 0$$

Boundary Geometry III

- The vielbein postulate constraints $\Gamma^c_{(0)ab}$ such that:

$$\left(\delta^d_b+\tau_{(0)b}\tau^d_{(0)}\right)\left(\delta^e_c+\tau_{(0)c}\tau^e_{(0)}\right)\nabla_{(0)a}\Pi_{(0)de}=0 \qquad \Pi_{(0)ab}=\delta_{\underline{ij}}e^{\underline{i}}_{(0)a}e^{\underline{j}}_{(0)b}$$

- We define $\Gamma^c_{(0)ab}$ by $\hat{\Gamma}^c_{(0)ab}=\Gamma^c_{(0)ab}$ for AlLif space-times but we keep this also for the Lifshitz UV completion so that:

$$\Gamma^c_{(0)ab}=-\frac{1}{2}\tau^c_{(0)}\left(\partial_a\tau_{(0)b}+\partial_b\tau_{(0)a}\right)+\frac{1}{2}\Pi^{cd}_{(0)}\left(\partial_a\Pi_{(0)bd}+\partial_b\Pi_{(0)ad}-\partial_d\Pi_{(0)ab}\right)$$
$$-\frac{1}{2}e^{3\Phi_{(0)}/2}\Pi^{cd}_{(0)}\left(F_{(0)da}\tau_{(0)b}+F_{(0)db}\tau_{(0)a}\right) \qquad \Pi^{ab}_{(0)}=\delta^{\underline{ij}}e^a_{(0)\underline{i}}e^b_{(0)\underline{j}}$$
$$F_{(0)ab}=\partial_aA_{(0)b}-\partial_bA_{(0)a}$$

- When $\partial_a\tau_{(0)b}-\partial_b\tau_{(0)a}=0$ the boundary geometry is Newton-Cartan for which we have $\nabla_{(0)a}\tau_{(0)b}=\nabla_{(0)a}\Pi^{bc}_{(0)}=0$.

Torsional Newton-Cartan

- With our $\Gamma^c_{(0)ab}$ obtained from the null reduction and introducing the torsion tensor $T^c_{(0)ab} = -\frac{1}{2}\tau^c_{(0)}\left(\partial_a\tau_{(0)b} - \partial_b\tau_{(0)a}\right)$ we get

$$\nabla^T_{(0)a}\tau_{(0)b} = \nabla_{(0)a}\tau_{(0)b} - T^c_{(0)ab}\tau_{(0)c} = 0$$

$$\nabla^T_{(0)a}\Pi^{bc}_{(0)} = \nabla_{(0)a}\Pi^{bc}_{(0)} + T^b_{(0)ad}\Pi^{dc}_{(0)} + T^c_{(0)ad}\Pi^{bd}_{(0)} = 0$$

- When $\tau_{(0)[a}\partial_b\tau_{(0)c]} = 0$ we call the torsion twistless because $\Pi^c_{(0)a}\Pi^d_{(0)b}\left(\partial_a\tau_{(0)b} - \partial_b\tau_{(0)a}\right) = 0$ where $\Pi^c_{(0)a} = \delta^c_a + \tau_{(0)a}\tau^c_{(0)}$

- TNC is obtained by dimensional reduction along a null circle generated by a null Killing vector.

I). $\tau_{[(0)a}\partial_b\tau_{(0)c]} = 0$ II). $\Phi_{(0)} = \text{cst}$ III). $\partial_a\tau_{(0)b} - \partial_b\tau_{(0)a} = 0$

	I	II	III	Boundary Geometry
ALif	Yes	Yes	Yes	NC
AlLif	Yes	Yes	No	TTNC
gen. AlLif	Yes	No	No	TTNC
Lif UV	No	No	---	TNC

- NC=Newton-Cartan TNC=torsional Newton-Cartan TTNC=twistless torsional Newton-Cartan

- Sources: $\tau_{(0)a}$ $e^{\underline{i}}_{(0)a}$ (bdry frame), $\chi_{(0)}$ (bdry axion), $\phi_{(0)}$ $\Phi_{(0)}$ boundary gauge field $A_{(0)}$ (leading term of $A_a - e^{-3\Phi/2}e^{\underline{t}}_a$)

Ward Identities

- On-shell variation of the action:

$$\delta S \sim \int d^3x e_{(0)} \Big(-S^{\underline{t}}_{(0)a} \delta\tau^a_{(0)} + S^{\underline{i}}_{(0)a} \delta e^a_{(0)\underline{i}} + T^{\underline{t}}_{(0)} \delta A_{(0)\underline{t}} + T^{\underline{i}}_{(0)} \delta A_{(0)\underline{i}}$$
$$+ \langle O_\Phi \rangle \delta\Phi_{(0)} + \langle O_\phi \rangle \delta\phi_{(0)} + \langle O_\chi \rangle \delta\chi_{(0)} - \mathcal{A}_{(0)} \frac{\delta r}{r} \Big)$$

- Local symmetries:
  - diffeomorphisms
  - anisotropic Weyl: $\delta\tau^a_{(0)} = 2w_{(0)}\tau^a_{(0)}\,,\ \delta e^a_{(0)\underline{i}} = w_{(0)} e^a_{(0)\underline{i}}$ $\qquad \delta r = w_{(0)} r$
    $\delta A_{(0)\underline{t}} = 2w_{(0)} A_{(0)\underline{t}}\,,\ \delta A_{(0)\underline{i}} = w_{(0)} A_{(0)\underline{i}}$
  - gauge: $\delta A_{(0)a} = \partial_a \Lambda_{(0)}\,,\ \ \delta\chi_{(0)} = k\Lambda_{(0)}$
  - local tangent space
- Only at leading order:

$$\delta\tau^a_{(0)} = -\omega_{(0)}\tau^a_{(0)}\,,\ \delta e^a_{(0)\underline{i}} = \omega_{(0)} e^a_{(0)\underline{i}}$$
$$\delta A_{(0)\underline{t}} = -\omega_{(0)} A_{(0)\underline{t}}\,,\ \delta A_{(0)\underline{i}} = \omega_{(0)} A_{(0)\underline{i}}$$
$$\delta\Phi_{(0)} = -2\omega_{(0)}$$

- The local tangent space rotation and gauge invariant boundary stress-energy tensor:

$$\mathcal{T}^b_{(0)a} = -\tau^b_{(0)}\left(S^{\underline{t}}_{(0)a} + T^{\underline{t}}_{(0)}\frac{1}{k}\partial_a\chi_{(0)}\right) + e^b_{(0)\underline{i}}\left(S^{\underline{i}}_{(0)a} + T^{\underline{i}}_{(0)}\frac{1}{k}\partial_a\chi_{(0)}\right)$$

- Ward identities for $\mathcal{T}^b_{(0)a}$ assuming $\partial_a\tau_{(0)b} - \partial_b\tau_{(0)a} = 0$

$$\nabla_{(0)b}\mathcal{T}^b_{(0)a} = -\mathcal{T}^c_{(0)b}\left(-\tau_{(0)c}\nabla_{(0)a}\tau^b_{(0)} + e^{\underline{i}}_{(0)c}\nabla_{(0)a}e^b_{(0)\underline{i}}\right)$$
$$-T^{\underline{t}}_{(0)}\partial_a B_{(0)\underline{t}} - T^{\underline{i}}_{(0)}\partial_a B_{(0)\underline{i}} - \langle O_\Phi\rangle\partial_a\Phi_{(0)} - \langle O_\phi\rangle\partial_a\phi_{(0)}$$
$$\mathcal{A}_{(0)} = -2\tau_{(0)b}\tau^a_{(0)}\mathcal{T}^b_{(0)a} + e^{\underline{i}}_{(0)b}e^a_{(0)\underline{i}}\mathcal{T}^b_{(0)a} + 2T^{\underline{t}}_{(0)}B_{(0)\underline{t}} + T^{\underline{i}}_{(0)}B_{(0)\underline{i}}$$
$$B_{(0)a} = A_{(0)a} - \frac{1}{k}\partial_a\chi_{(0)}$$

- Similar to HIM boundary stress tensor [Hollands, Ishibashi, Marolf, 2005]. For conserved boundary currents sufficient to set $\mathcal{L}_{K_{(0)}}\tau^a_{(0)} = \mathcal{L}_{K_{(0)}}\Pi^{ab}_{(0)} = \mathcal{L}_{K_{(0)}}\text{bdry scalar} = 0$ so $\nabla_{(0)b}\left(K^a_{(0)}\mathcal{T}^b_{(0)a}\right) = 0$ These are then of the type discussed in [Ross, Saremi, 2009].

Lifshitz scale anomaly

- For ALif boundary conditions the anomaly is

$$\frac{2\pi L}{\kappa_5^2}\mathcal{A}_{(0)} = C_1\left(4K_{(0)ab}K^{ab}_{(0)} - 2K^2_{(0)}\right) + C_2\mathcal{R}^2_{(0)} + \nabla_{(0)a}J^a_{(0)}$$

- $K_{(0)ab}$ is the extrinsic curvature and $\mathcal{R}_{(0)}$ is a 4th order derivative term. Both are invariant under anisotropic conformal rescalings.

- $\mathcal{A}_{(0)}$ is of the form of a z=2 conformally invariant 3D Horava-Lifshitz action. These can thus be defined even if there is no Lorentzian metric.

- In general there are two central charges [Baggio, de Boer, Holsheimer, 2011],[Griffin, Horava, Melby-Thompson, 2011]. Here they are $C_1 = 6C_2 = \dfrac{l^2_{\text{Lif}}}{64\pi G_4}$ [Chemissany, Geissbuehler, JH, Rollier, 2012].

Counting Sources & Vevs

- #sources=15(components)-8(local symmetries) -1(constraint)=6
- #vevs=15(components)-9(Ward identities including the one for the leading order symmetry)=6
- In the Fefferman-Graham expansion there appears another function $\tau^a_{(0)}\tau^b_{(0)}\hat{t}_{ab}$ at order r^2 that does not appear in the Ward identities.
- We thus have 6+6+1 free functions in the expansion.
- In 5D this function is a certain component of the boundary stress-energy tensor that drops out of the Ward identity upon reduction.

Summary

- There exists a Lifshitz model that is a well-defined low energy limit of IIB string theory.
- Found all deformations of AlLif space-times.
- Frame fields useful for boundary geometry (torsional Newton-Cartan) and coordinate independent definition of boundary stress-energy tensor.
- There are 6+6 sources & vevs and one additional free function that control the Fefferman-Graham expansion.
- Interpretation of the extra free function?

Outlook

- Bottom-up models: what about the massive vector model (no scalars)? Do they display the extra free function?
- Metric expansion in radial gauge.
- Different values of z?
- Adding hyperscaling violation, e.g. this model also contains $\theta = 1$ and $z = 3$.
- Adding charge.
- Applications to CMT: Son's model for fractional quantum Hall effect [Son, 2013] which uses NC geometry.