Hurricane dynamics: on the role of Vortex Rossby Waves (VRWs).

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Newton Institute, 28 October 2013

Acknowledgements: Y. Chen, Y. Martinez, K. Menelaou and P. Yau,
Outline

Introduction

• The challenge of numerical weather prediction: hurricanes, typhoons and tropical cyclones

• Hurricane intensity: on the predictability and dynamical processes
  • Empirical Normal Mode Analysis
  • Unstable vortex: Diagnostics of 2D barotropic VRW numerical experiments
  • Eyewall replacement cycle: Diagnostics of 3D numerical experiments

• Conclusion
The challenge of predicting hurricanes

Hurricane climatology: a concern for North American.

Hurricane Juan, 28 September 2003, Halifax
GLOBAL Environment Canada NWP System GEM (HR: 35km)
Instantaneous precipitation rate (mm/hr) for the Operational GEM model
A 5 day animation (20/01/2002 to 25/01/2002) (HR=100km, TR=45 min.)

Acknowledgement to M. Roch and S. Bélair
Instantaneous precipitation rate (mm/hr) for the Meso-Global GEM model
A 5 day animation (20/01/2002 to 25/01/2002) (HR=33km, TR= 15 min.)

Acknowledgement to M. Roch and S. Bélair
DYNA Cyclone: cat. 4 - 130knts
Intensification of tropical cyclones: a multi-scale dynamical and physical process

The tropical cyclone Dina at 100km horizontal resolution

The tropical cyclone Dina at 33 km horizontal resolution
On the dynamics of hurricanes
Motivation and Background

- Andrews (1992)
  Spiral rainbands have been reasonably well explained by invoking vortex Rossby waves (VRWs) Dynamics. As VRWs radiate outward, cyclonic angular momentum is transported inward to the parent vortex producing vortex intensification. (Vortex Symmetrization Mechanism. Montgomery and Kallenbach, 1997)
Motivation and Background

- Isabel (2003)

Mesovortices and polygonal eyewalls can be explained as a result of VRW instabilities. Then the mature hurricane rapid intensification occurs via eyewall contraction. (*Schubert, 1999*)
Can we characterize and quantify the dynamics and significance of the spiral bands?

Studies have shown that inner spiral bands have characteristics of vortex Rossby-waves.

Vortex Rossby-waves (VRW) and gravity waves frequencies are mixed (Rossby number \([U/Lf]\) is not small).

Apply Empirical Normal Mode (ENM) method to separate the waves to isolate the effect of VRW on a simulated hurricane (6 km grid size, 24 h simulation sampled every 2 minutes).

Empirical Normal Mode (ENM) analysis

- principal component (EOF) analysis
- takes advantage of wave activity conservation law
- decomposes simultaneously wind and thermal fields into dynamically consistent and orthogonal modes with respect to wave activities
- used to study Rossby waves (Brunet 1994, Brunet and Vautard 1996, Zadra et al. 2002), gravity waves (Charron and Brunet 1999) and VRWs (Chen et al., 2003, Martinez et al., 2010, 2011)
Derivation of wave activity equation

- Hydrostatic primitive equations in storm-following isentropic coordinates
- Assume small amplitude perturbations
- Basic state = time-azimuthal mean
- Azimuthal-invariant → pseudo-momentum density conservation
- Time-invariant → pseudo-energy density conservation
Derivation of wave activity equation (cont.):

Pseudo-momentum equation:

\[
\frac{\partial J}{\partial t} + \frac{\partial F_\lambda}{r \partial \lambda} + \frac{\partial r F_r}{r \partial r} + \frac{\partial F_\theta}{\partial \theta} = S_J
\]

\[J = -r \sigma' v' \frac{r \overline{\sigma}^2 q'^2}{2\gamma}
\]

\[\gamma = \frac{\partial q}{\partial r}
\]

\[F_\lambda = -\bar{v} J - \frac{r \overline{\sigma}}{2} (v'^2 - u'^2) - \frac{rR}{2gp} \left( \frac{\bar{p}}{p_s} \right)^\kappa p'^2
\]

\[F_r = -ru' v' \overline{\sigma} \quad \sim -u' v' \text{ inward momentum trans.}
\]

\[F_\theta = \frac{p'}{g} \frac{\partial M'}{\partial \lambda} \quad \sim -u' T' \text{ inward heat trans.}
\]
Derivation of wave activity equation (cont.): Pseudo-energy equation:

\[
\frac{\partial A}{\partial t} + \frac{\partial E_\lambda}{r \partial \lambda} + \frac{\partial r E_r}{r \partial r} + \frac{\partial E_\theta}{\partial \theta} = S_A
\]

\[
A = \frac{\bar{\sigma}}{2} (u'^2 + v'^2) + \frac{R}{2g\bar{p}} (\frac{\bar{p}}{p_s})^\kappa p'^2 - \frac{\bar{v}J}{r}
\]

- **KE** \( E_\lambda = \frac{\bar{\sigma}}{2} \bar{v}v'^2 + \bar{\sigma}v'M' + \sigma'\bar{v}M' - \frac{\bar{v}^2}{r} J \)

- **PE** \( E_r = \frac{\bar{\sigma}}{2} \bar{v}u'v' + \bar{\sigma}u'M' \)

- **DS** \( E_\theta = -\frac{M'}{g} \frac{\partial p'}{\partial t} \)
EOF ~ ENM

- **ENM**

\[
Z(t, x) = \sum_n a_n(t)F_n(x) + \varepsilon
\]

\[
C(t_1, t_2) = \left\langle Z^T(t_1, x)B(x)Z(t_2, x) \right\rangle
\]

\[
C \cdot a_n = \lambda_n a_n
\]

\[
F_n(x) = Z(t, x)a_n(t)
\]

### Matrix Notation

\[
Z^T = (u' \, v' \, p' \, \sigma' \, q')
\]

\[
B = -\frac{r}{2}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\bar{\sigma}^2}{\gamma}
\end{bmatrix}
\]

- The norm of C and \( \lambda_n \) are pseudo-momentum
- ENM extracts modes which conserve wave activities (J and A) when dynamics is linear
Empirical Normal Mode (ENM)

- The ENM theory bridges two important diagnostic tools for geophysical fluid dynamics studies: principal component analysis and normal mode theory.

\[
T_n = \frac{2\pi}{s} \left| \frac{J_n}{A_n} \right|
\]

\[
c_n = -\frac{A_n}{J_n} = \frac{\bar{v}}{r} - \left[ \frac{KE + PE}{J} \right]_n,
\]

- The ENM phase speed formula provides the centroid of the frequency power spectrum when the linear equation is stochastically forced.
Model data analysis

• Procedures
  • vertical interpolation of variables from model coordinates to $\theta$ isentropic coordinates
  • remove basic state (time-azimuthal mean)
  • azimuthal decomposition
  • find ENM modes (time series + spatial patterns)
  • discuss wave modes kinematics and dynamics
PV at 6 km

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Basic State

\[ \gamma > 0 \]

\[ \gamma < 0 \]
Wavenumber 1 and 2, ENM mode J and A spectra

- J and A change sign
- Period and phase speed

\[
T_n = \frac{2\pi}{s} \left| \frac{J_n}{A_n} \right|
\]

\[
c_n = \frac{-A_n}{J_n} = \frac{\bar{v}}{r}
\]

- Retrograde and prograde waves
- No unstable mode because the azimuthal wind profile is monotonic (Ren, 2003)
Wavenumber 1 and 2, ENM mode J and A spectra

- Rotational contribution > gravitational contribution
- Leading modes are vortex Rossby waves
- Spiral rainbands show vortex Rossby wave characteristics
Azimuthally propagating waves

\[ Z = \text{Re}[Ae^{i(s\lambda - \omega t)}] \]

\[ = [\text{Re}(A) \cos s\lambda - \text{Im}(A) \sin s\lambda] \cos \omega t \]

Mode 1

\[ + [\text{Im}(A) \cos s\lambda + \text{Re}(A) \sin s\lambda] \sin \omega t \]
Wavenumber 2, ENM mode 1 and 2, time series

\[ T_{1,2} = \frac{2\pi}{\text{s}} \left| \frac{J_{1,2}}{A_{1,2}} \right| \]

\[ \sim 1.1\text{h} \]

\[ \Delta T = 0.27\text{h} \]

\[ T = 1\text{h} \]

\[ |a| \sim 2 \]

\[ 2\pi/(24\text{H}) \]
Wavenumber 2, PV ENM mode 1 and 2, spatial patterns
Wavenumber 1, ENMs average frequencies

- Continuous spectrum
- Discrete spectrum

$\Omega_{\text{max}} \left( \frac{2\pi}{24H} \right)$
Period and variance (%) of most important ENMs that contribute to 47% of the total variance

<table>
<thead>
<tr>
<th>Wave number</th>
<th>ENM number</th>
<th>Period (hour)</th>
<th>Variance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2.4</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2.4</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>720</td>
<td>1.6</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>721</td>
<td>1.6</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.0</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>720</td>
<td>1.1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>721</td>
<td>1.1</td>
<td>3</td>
</tr>
</tbody>
</table>

- Numerical Weather Prediction (NWP) or regional climate models with timestep less than one hour should start to resolve properly Vortex Rossby Waves (VRWs)

NB: Gravity wave variance negligible
Wavenumber 1 and 2, mode 1+2, EP flux (24h time mean)

• Wave-mean-flow interaction:

\[
\frac{\partial \left( r \bar{\sigma} \bar{v} \right)}{\partial t} + \frac{\partial}{\partial r} \left( r^2 \bar{\sigma} \bar{u} \bar{v} \right) + (r \bar{\sigma} \bar{u}) f = \nabla \cdot \vec{F} - \frac{\partial \left( r \sigma' v' \right)}{\partial t}
\]

1~2 m/s per hour

• Acceleration: low and middle troposphere inside/outside the eyewall
• Deceleration: upper troposphere in the eyewall
• Critical layer signature

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Numerical simulations of a vorticity ring

- **NUB model**
  - Time step (7.5 sec)
  - Time sampling (every 2.5 min)
  - Total time (6h)
  - Domain (600 x 600 Km)
  - Output domain (175 x 175 Km)
  - Resol. (1.17km)

- **Initialization** - Ring+ broadbanded noise

*Schubert el. al (1999)*
Basic state (Ring)

$V_{tMax} \sim 52 \text{ m/s}, \ RMW \sim 60\text{km}$

$W_{max} \sim 0.0011 \text{ s}^{-1}$

$\xi_0 \sim 0.0035 \text{ s}^{-1}$

Max $\sim 0.003 \text{ s}^{-1}$
Vorticity anomaly

vorticity anomaly x 1.e-3 (1/s) T=3H

vorticity anomaly x 1.e-3 (1/s) T=4H
Wave Activity spectra

- retrograde + prograde waves are important
- pseudomomentum \((J)\) expressed only by a vortical term
- unstable modes may arise (gamma changes sign)
- \(K > DS\)
ENM complex patterns

(a) Wavenumber 4 mode 1 cos

(b) Wavenumber 4 mode 1 sin

(c) Wavenumber 4 mode 2 cos

(d) Wavenumber 4 mode 2 sin
ENM complex patterns

(a) Wavenumber 4 mode 144 cos

(b) Wavenumber 4 mode 144 sin

(c) Wavenumber 4 mode 145 cos

(d) Wavenumber 4 mode 145 sin
• Total azimuthal wind acceleration and deceleration inside the RMW (at 60 km)

• Both prograde and retrograde waves contribute to the EP flux div

• Modes 144+145 are prograding and transfer positive angular momentum towards the eyewall
# Frequencies and Periods

<table>
<thead>
<tr>
<th>Mode#</th>
<th>VAR (%)</th>
<th>Periods (H)</th>
<th>Freq. (.001)</th>
<th>Cross-Correlation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>56</td>
<td>0.55</td>
<td>3.17</td>
<td>- 99.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>99.55</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>0.50</td>
<td>3.43</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td>4.8</td>
<td>0.530</td>
<td>3.28</td>
<td>99.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>- 99.52</td>
</tr>
<tr>
<td>145</td>
<td>4.9</td>
<td>0.534</td>
<td>3.25</td>
<td></td>
</tr>
</tbody>
</table>
As a hurricane intensifies a secondary eyewall (SE) forms concentrically outside the primary eyewall.

- The SE propagates inward and intensifies, the inner eyewall weakens and is eventually replaced by the outer eyewall in a process known as Eyewall Replacement Cycle (ERC).

- About 70% of all major hurricanes have undergone ERC. SE and ERC are usually associated with significant intensity fluctuations.

- SE and ERC are vital to understand inner-core dynamics, upgrading land-falling warnings and improve the intensity forecasting.
On the Dynamics of Concentric Eyewall Genesis

- Hurricane Wilma (2005) is simulated using the Weather Research and Forecast (WRF) model.

- The Empirical Normal Mode (ENM) method is used to extract the dominant wave modes from the data set.

- Eliassen-Palm (EP) flux is used to diagnose the impact of these wave modes on the formation of the secondary eyewall.
Basic set up of the model

- Four domains (27, 9, 3 and 1 km) with two way interaction and a movable nest
- Geophysical Fluid Dynamics Laboratory (GFDL) initial and boundary conditions
- NCEP real time global sea surface temperature (SST) with 0.5° resolution
- Initialized at 0000 UTC 18 Oct., 2005 and run for 72 hours.

<table>
<thead>
<tr>
<th>Domains</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal grid mesh</td>
<td>140 x 140</td>
<td>286 x 286</td>
<td>484 x 484</td>
<td>346 x 346</td>
</tr>
<tr>
<td>Grid spacing</td>
<td>27km</td>
<td>9km</td>
<td>3km</td>
<td>1km</td>
</tr>
<tr>
<td>Vertical levels</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Cumulus parameterization</td>
<td>Betts-Miller</td>
<td>Betts-Miller</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Microphysics</td>
<td>Thompson</td>
<td>Thompson</td>
<td>Thompson</td>
<td>Thompson</td>
</tr>
<tr>
<td>Planetary boundary layer</td>
<td>Mellor-Yamada</td>
<td>Mellor-Yamada</td>
<td>Mellor-Yamada</td>
<td>Mellor-Yamada</td>
</tr>
</tbody>
</table>
Model domain configuration (Di & Df Resolution is 1km)
Model’s performance in simulating hurricane Wilma

Simulated track (grey) and NHC 6-h best track analysis (black)

Central sea level pressure (hPa) from the model (grey line) and from NHC (circles)
Simulated secondary eyewall formation and eyewall replacement cycle of hurricane Wilma

Simulated reflectivity (dBZ) at 2km height

Azimuthal average tangential winds (m/s)
Basic state (time & azimuthal average)

Angular velocity (s⁻¹)  |  Potential vorticity (PVU)  |  PV gradient (PVU/km)

(a)  |  x 10⁻³  |  (c)
Wave activity spectra

Rotational contribution (VRWs) to pseudomomentum

Gravitational contribution to pseudomomentum

- Rotational contribution (VRW) > Gravitational contribution
- Leading modes are Vortex Rossby Waves

Phase speed,

\[ C_n = -\frac{A_n}{J_n} = \frac{v_0}{r} - \frac{K + P}{J} \]

Intrinsic phase speed

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Power spectra and ENM mode 1 and 2

- Same frequency of oscillation
- High cross correlation among their spatial patterns (-85% and 79%)

Mode 1 and 2 form a retrograde VRW
EP flux and EP flux divergence for ENMs 1 and 2

Horizontal EP flux (\(kgmK^{-1}s^{-2}\))

EP flux divergence (\(kgK^{-1}s^{-2}\))

- \(-\cdots\cdots\) = Critical radius
  \[
  \omega = m\Omega_0(r^*)
  \]

- EP-flux maximum coincide with the critical radius
- Acceleration of the mean tangential wind at the location where secondary eyewall eventually forms
ENM analysis during a period when there is NO secondary eyewall formation

No maximum in the EP-flux divergence outside the primary eyewall
EP flux divergence calculated directly from the WRF output (312 K isentrope)

Strong signal close to the critical radius predicted by the ENM analysis
Conclusion

- A Vortex Rossby Wave (VRW) wave-mean flow interaction mechanism is proposed for hurricane dynamics to explain the intensification of primary wall and the eye wall replacement cycle

- Consequence for forecast skill:
  - VRWs need to be properly simulated to obtain correct momentum redistribution (e.g. wave breaking) inside hurricanes and hence intensification evolution
  - We expect the numerical convergence of momentum flux at convective scale (~ 1km)

- Theoretical challenge: the eyewall replacement cycle is a critical layer problem with a moving critical line. An analytical study could help a lot our understanding of this dynamical process.
References


The forced primitive equations in storm-following cylindrical and isentropic coordinates \((r, \lambda, \theta)\) are

\[
\begin{align*}
\frac{\partial}{\partial t} u + \frac{1}{2} u^2 + \frac{1}{2} v^2 + M \frac{\partial}{\partial r} &= F - \frac{\partial}{\partial \lambda} \theta, \quad (2a) \\
\frac{\partial}{\partial t} v + \frac{1}{2} u^2 + \frac{1}{2} v^2 + M \frac{\partial}{\partial \lambda} &= \eta u = G - \frac{\partial}{\partial \theta} \theta, \quad (2b) \\
\sigma_t + \frac{1}{r} (\sigma u)_{\lambda} + \frac{1}{r} (r \sigma u)_r &= -(\eta \theta)_{\theta}, \quad (2c) \\
M_\theta &= C_p \left( \frac{p}{p_s} \right)^\kappa, \quad (2d) \\
p_\theta &= -g \sigma, \quad (2e)
\end{align*}
\]

where \(u\) and \(v\) are, respectively, the radial wind and tangential wind relative to the moving hurricane center.
Hurricane prediction: the present and future
"The major model guidance they (on the forecast desks) used were the ensembles for their consistency and overall agreement, especially the ensemble means from ALL the centers (NCEP, ECMWF, CMC, UKMET) ……. I believe the consistency of the message was key to making the impact that it did…..to convince the emergency management community and others to pay attention and take action."

Lois Uccellini, Director US National Center Environmental Prediction

- Largest Atlantic hurricane on record
- 2nd costliest Atlantic hurricane (after Katrina)
- At least $20bn damage, $50bn total cost including business interruption
- At least 185 deaths
As Sandy was forming (7 days before US landfall) the strong probability of US landfall was flagged by MOGREPS.

Highest strike probabilities north of actual track (shown in black).
4 days before US landfall MOGREPS gave a good indication of landfall location.
4km Model Forecast of Hurricane Sandy
A Grand Challenge project on the Earth Simulator: A Japan-Canada collaboration

Relative Vorticity at 950 hPa (10km)
Humidity at 350m height is shown over Gulf of Mexico for the first 12-72 hours of the simulation.
-Only 1% of the simulation domain is shown!

- 10000 km X 10000 km X 75 vertical levels
- space resolution: 1km
- sustained performance: 13TFlops
Thank you!

Merci!