Direct Statistical Simulation
of a
Two-Layer Primitive Equation Model

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DSS vs. DNS

Low-order statistics are smoother in space than the instantaneous flow.

Correlations are *non-local* and highly anisotropic and inhomogeneous.

Statistics evolve slowly in time, or not at all, and hence may be described by a fixed point.
Mean Flows and Eddies

\[ q = \langle q \rangle + q' \]

Eddies sheared apart by mean flows
A Conservative Approximation

CE2 conserves total angular momentum, energy, and enstrophy in absence of forcing and damping.

CE3 additionally conserves the 3rd Casimir.

Realizability

CE2 = Quasilinear = Gaussian PDF = Realizable

CE3: Not realizable. Fix this by projecting out negative eigenvalues of second-cumulant [Kraichnan (1980)].
Zonal Averages

\[ \langle q_{l_1 m_1} q_{l_2 m_2} q_{l_3 m_3} \rangle \]

\[ \langle q_{l_1 m} q_{l_2 m} q_{l_3,0} \rangle = \langle q_{l_1 m} q_{l_2 m} \rangle \langle q_{l_3,0} \rangle = c_{l_1 l_2 m} c_{l_3} \]

CE2 : \[ \langle q_{l_1 m_1} q_{l_2 m_2} q_{l_3 m_1+m_2}^* \rangle = 0 \text{ if } m_2 > 0 \text{ and } m_1 > 0 \]

\[ \dot{c}_l = A_l + B_{l; l_1 0} c_{l_1} + C_{l; l_1 m; l_2 m}^{(-)} c_{l_1 l_2 m} \]

\[ \dot{c}_{l_1 l_2 m} = 2 \Gamma_{l_1 m} \delta_{l_1 l_2} + B_{l_1; l m} c_{l l_2 m} + B_{l_2; l m} c_{l_1 l m} \]

\[ + C_{l_1; l 0; l' m} c_{l} c_{l l_2 m} + C_{l_2; l 0; l' m} c_{l} c_{l_1 l' m} \]
Demonstration:
Hydrodynamic Shear
GCM

By Brad Marston

Open the Mac App Store to buy and download apps.

Description

Idealized General Circulation Models (GCMs) of planetary atmospheres, solved by a variety of methods.

GCM Support

What’s New in Version 1.0.4

New wave lifecycle model, better organized menu. Bug fixes to CE3 (now conserves 3rd Casimir) and the calculation of the eddy diffusivity.

Screenshots

Free
Category: Education
Updated: May 23, 2013
Version: 1.0.4
Size: 1.4 MB
Language: English
Seller: Brad Marston
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Rated 4+

Requirements: OS X 10.8.3 or later, 64-bit processor

Customer Ratings

We have not received enough ratings to display an average for the current version of this application.
All Versions: 8 Ratings
Two-layer primitive equations: Relaxation to a prescribed equator-to-pole temperature difference (Held-Suarez)
\[ \vec{v} = \vec{r} \times \nabla \psi + \nabla \chi, \]

\[ J[A, B] \equiv \vec{r} \cdot (\nabla A \times \nabla B) \]

\[ F[A, B] \equiv \nabla \cdot (A \nabla B), \]

\[ \dot{q} = J[\bar{q}, \bar{\psi}] + J[\hat{q}, \hat{\psi}] - F[\hat{q}, \hat{\chi}] - J[\hat{\delta}, \hat{\chi}] - F[\hat{\delta}, \hat{\psi}], \]

\[ \dot{q} = J[\hat{q}, \hat{\psi}] + J[\bar{q}, \bar{\psi}] - F[\bar{q}, \bar{\chi}], \]

\[ \dot{\delta} = J[\bar{q}, \bar{\chi}] + F[\hat{q}, \hat{\psi}] + F[\bar{q}, \hat{\psi}] - \nabla^2 (\hat{K} + C_p B \hat{\theta}), \]

\[ \dot{\theta} = J[\bar{\theta}, \bar{\psi}] + J[\hat{\theta}, \hat{\psi}] - F[\hat{\theta}, \hat{\chi}], \text{ and} \]

\[ \dot{\theta} = J[\hat{\theta}, \hat{\psi}] + J[\bar{\theta}, \bar{\psi}] - F(\bar{\theta}, \bar{\chi}) + \bar{\theta} \delta. \]
Demonstration:
Primitive Equations
Quasi-linear:

Quasi-linear + Small Stochastic:

CE2:
Observation

(Courtesy F. Sabou)
Quasi-linear:

Quasi-linear + Small Stochastic:

CE2:
Spectra (barotropic vorticity)

481.7 days

DNS

2000.0 days

CE2

600.0 days

CE3
"More than any other theoretical procedure, numerical integration is also subject to the criticism that it yields little insight into the problem. The computed numbers are not only processed like data but they look like data, and a study of them may be no more enlightening than a study of real meteorological observations. An alternative procedure which does not suffer this disadvantage consists of deriving a new system of equations whose unknowns are the statistics themselves...."


“Direct Statistical Simulation” (DSS)