Robust Engineering of Correlations and Symmetries on Quantum Networks

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Design of Open Quantum Dynamics

➡ **Task:** Develop accurate and ***systematic*** methods for *control and design of open quantum dynamics*, inspired by control theory's approach.

➡ **Fundamental & Practical Motivation:**

✓ Robust realizations of Quantum Information Processing (QIP) - *Viola's Talk;*

✓ Enhanced sensitivity in precision measurements & metrology - *Plenio's Talk;*

✓ Engineer new "forms"/phases of matter;

- **Two Prevailing & Complementary Approaches:**

I. **Environment as Enemy: we want to "remove" the coupling.**
Noise suppression methods, active and passive, including hardware engineering, noiseless subsystems, quantum error correction, dynamical decoupling [e.g. *Li's Talk, Plenio's Talk*];

II. **Environment as Resource: we want to "engineer" the coupling.**
Needed for state preparation, open-system simulation, and much more...

Dissipation for Information Engineering

- **Dissipation for QIP**
- **Open System Simulator**
- **Entanglement Generation**

nature physics LETTERS
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Quantum computation and quantum-state engineering driven by dissipation

Frank Verstraete[1*], Michael M. Wolf[2] and J. Ignacio Cirac[3*]

ARTICLE
doi:10.1038/nature09801

An open-system quantum simulator with trapped ions

Julio T. Barreiro[1*], Markus Müller[2,3*], Philipp Schindler[1], Daniel Nigg[1], Thomas Monz[1], Michael Chwalla[1,2], Markus Hennrich[1], Christian F. Roos[1,2], Peter Zoller[2,3] & Rainer Blatt[1,2]

PRL 107, 080503 (2011) PHYSICAL REVIEW LETTERS week ending 19 AUGUST 2011

Entanglement Generated by Dissipation and Steady State Entanglement of Two Macroscopic Objects

Hanna Krauter,[1] Christine A. Muschik,[2] Kasper Jensen,[1] Wojciech Wasilewski,[1,*] Jonas M. Petersen,[1] J. Ignacio Cirac,[2] and Eugene S. Polzik[1,†]

LETTER
doi:10.1038/n

Deterministic entanglement of superconducting qubits by parity measurement and feedback

D. Ristè[1], M. Dukalski[1], C. A. Watson[1], G. de Lange[1], M. J. Tiggelman[1], Ya. M. Blanter[1], K. W. Lehnert[2], R. N. Schouten[1] & L. DiCarlo[1]

LETTER
doi:10.1038/nature12802

Autonomously stabilized entanglement between two superconducting quantum bits

S. Shankar[1], M. Hatridge[1], Z. Leghtas[1], K. M. Sliwa[1], A. Narla[1], U. Vool[1], S. M. Girvin[1], L. Frunzio[1], M. Mirrahimi[1,2] & M. H. Devoret[1]

Can I use dissipative dynamics to prepare and preserve *any* target state?

Simple state stabilization

Environment Engineering for a Single System

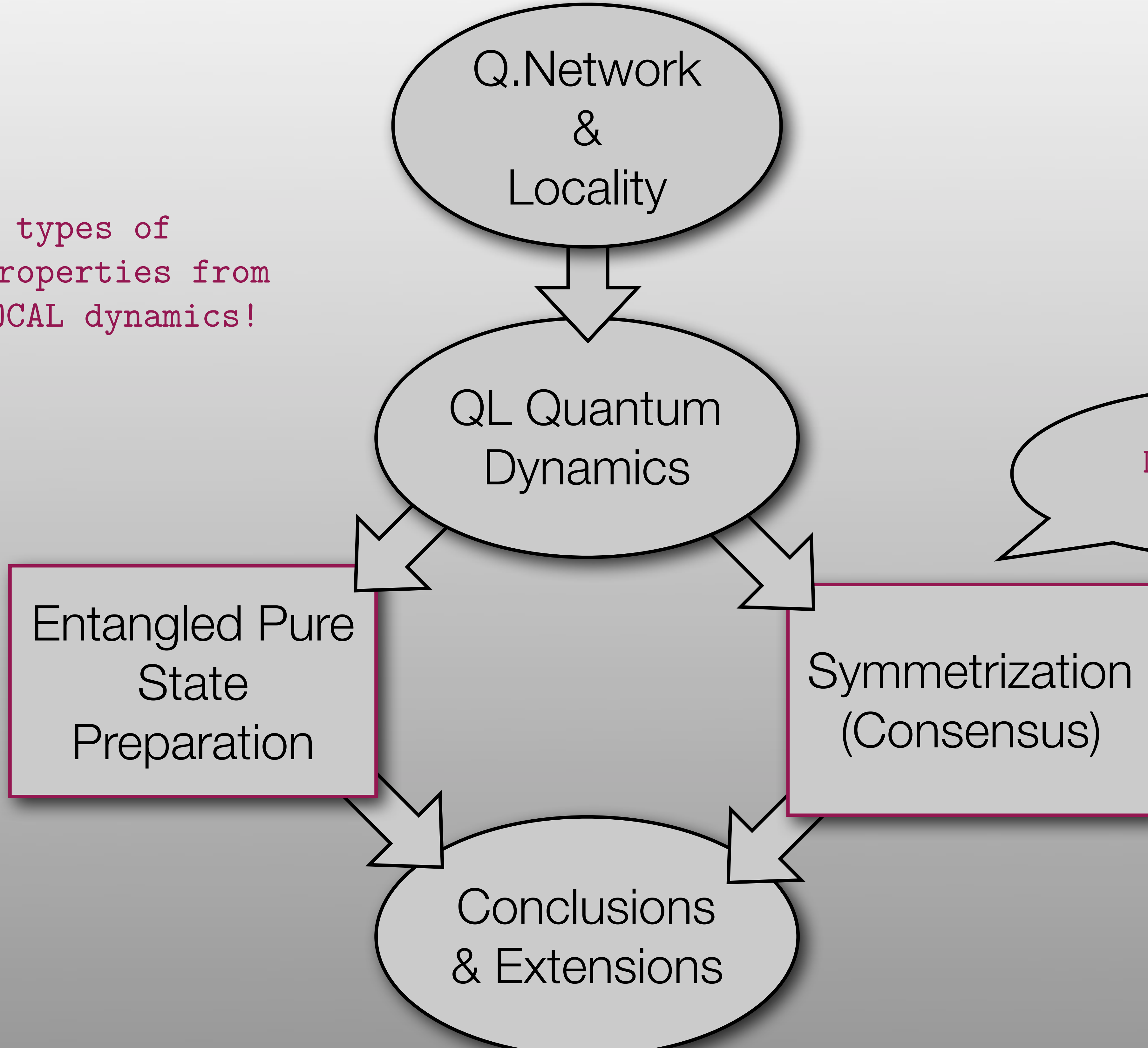
- **Naive Answer: YES!**
  A "simple" Markov evolutions that do the job always exists:
  - **Pure state:** e.g. Lindblad gen. with single L is enough, with ladder-type operator; [T-Viola, IEEE T.A.C., 2008]
    Similar results for discrete time; [Bolognani-T, IEEE T.A.C., 2010]
  - **Mixed state: e.g. also Lindblad with H and a single L (tri-diagonal matrices);** [T-Schirmer-Wang, IEEE T.A.C., 2010]

- **Can I obtain it with experimentally-available, constrained control capabilities? Typically not.**
- We need to take into account:
  - The control method [open-loop, switching, feedback, coherent feedback,...]
  - Limits on speed and strength of the control actions;
  - **Faulty controls, uncertain states,...**
  - **Locality constraints.**

key
limitation for
multipartite
entanglement
generation

Structure of the Talk

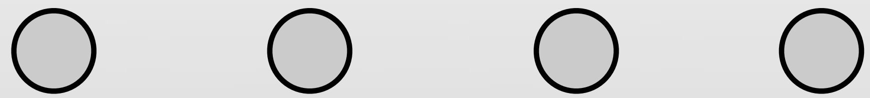
Two types of
global properties from
QUASI LOCAL dynamics!



Luca
Mazzarella's
Poster!

Multipartite Systems and Locality Constraints

- Consider *n* finite-dimensional systems, indexed:



$$a = 1 \quad 2 \quad 3 \quad \cdots$$

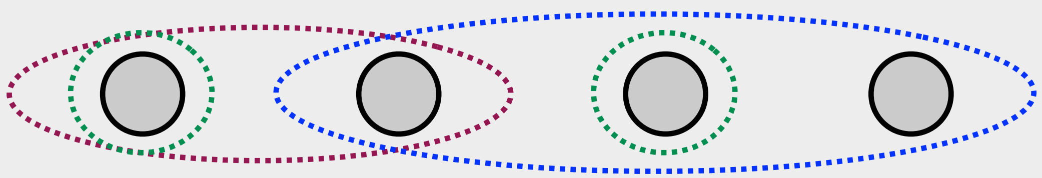
$$\mathcal{H}_\mathcal{Q} = \bigotimes_{a=1}^{n} \mathcal{H}_a$$

- **Locality notion:** from the start, we specify *subsets of indexes,* or ***neighborhoods,*** corresponding to group of subsystems:

$$\mathcal{N}_1 = \{1, 2\}$$

$$\mathcal{N}_2 = \{1, 3\}$$

$$\mathcal{N}_3 = \{2, 3, 4\}$$



...on which "**we can act simultaneously**": how?

- An *operator* is said Quasi-Local (QL) if $M_k = M_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$
- A *Hamiltonian* is said QL if:

$$H = \sum_k H_k, \quad H_k = H_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$$

Note: definitions depend on the neighborhoods!

This framework encompasses different notions: graph-induced locality, N-body locality, etc...

Quantum Discrete-Time Markov Dynamics

- We consider sequences of quantum channels [CPTP maps]:

$$\{\mathcal{E}_t\}_{t\geq 0}$$

- At each time step:

$$\rho_{t+1} = \mathcal{E}_t(\rho_t) = \sum_{k=1}^{p} M_{t,k}\rho_t M_{t,k}^\dagger$$

$$\sum_{k=1}^{p} M_{t,k}^\dagger M_{t,k} = I, \quad \forall t \geq 0$$

- We then constrain each of the $M_{k,t}$ to be QL with respect *to the same* neighborhood, $\mathcal{N}_t$. **The channel $\mathcal{E}_t$ is then said to be QL.**

Can I use QL dynamics
to prepare entangled pure states?

State engineering under locality constraints

~ discrete-time counterpart of

[T-Viola, *Phil. Trans. R. Soc. A, 2012; T-Viola, QIC, 2013*]

Quasi-Local Stabilizable State

- Assume (for now) that we can engineer **any QL channel**;

- **Definition:** A state $\rho = |\psi\rangle\langle\psi|$ is **Quasi-Local Stabilizable (QLS)** if there exist a **sequence of QL channels** $\{\mathcal{E}_t\}_{t\geq 0}$ such that:

I) $$\forall\, t \geq 0 \quad \mathcal{E}_t(\rho) = \rho$$ *[invariance]*

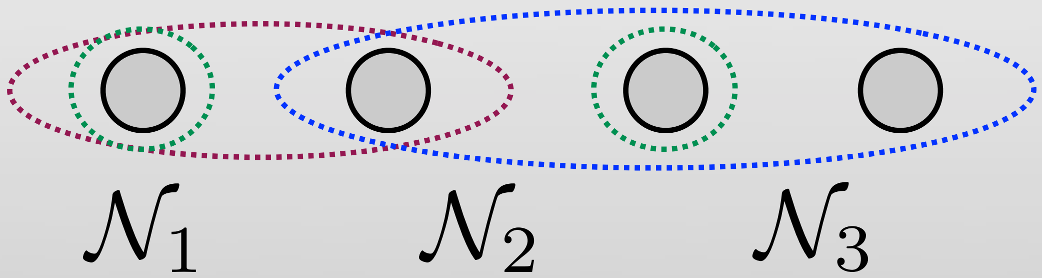
II) $$\forall\, \rho_0, \ \lim_{t\to\infty} \|\rho_t - \rho\| = 0$$ *[convergence]*

where: $$\rho_t = \mathcal{E}_t \circ \ldots \circ \mathcal{E}_1(\rho_0)$$

- *Remark: it ensures QL dynamics, and robustness w.r.t. to the initial state.*
- **Which states are QLS? Is there a way to test a target state?**
- **How to engineer locality-constrained operators that do the job?**

Test and Characterization of QLS

- For each *neighborhood* compute the reduced states;



$$\rho_{\mathcal{N}_1},\ \rho_{\mathcal{N}_2},\ \rho_{\mathcal{N}_3}$$
$$\rho_{\mathcal{N}_k} = \mathrm{trace}_{\bar{\mathcal{N}}_k}(\rho),$$

- For each neighborhood calculate the *support* of the reduced state times the identity on the rest:

$$\mathcal{H}_{\mathcal{N}_k} = \mathrm{supp}(\rho_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k})$$

- ***Theorem:***

$$\mathcal{H}_0 := \bigcap_k \mathcal{H}_{\mathcal{N}_k} = \mathrm{supp}(\rho)$$

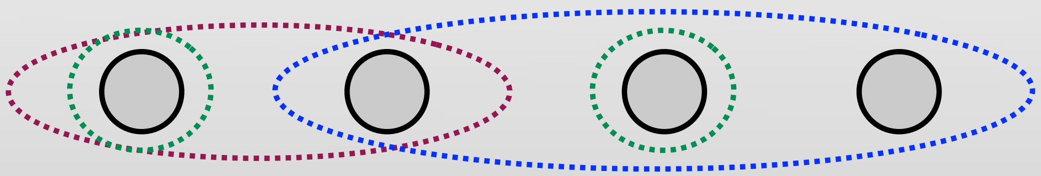
if and only if ρ is QLS;

Provides a simple, easily automated test with only two inputs: <u>the state and the neighborhoods</u>

IDEA: the support is “where the probability is”;
Locally I can only know the reduced state, and I try to prepare it.

Proof Sketch - 1

- For each *neighborhood* we can **"stabilize the support"** of the reduced state;



$$\rho_{\mathcal{N}_1},\ \rho_{\mathcal{N}_2},\ \rho_{\mathcal{N}_3}$$

$$\rho_{\mathcal{N}_k} = \mathrm{trace}_{\bar{\mathcal{N}}_k}(\rho),$$

i.e. $\mathcal{E}_{\mathcal{N}_k}$ stabilize the states with range in: $\mathcal{H}_{\mathcal{N}_k} = \mathrm{supp}(\rho_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k})$

- Consider a **cyclic evolution**:
  (K: number of neighborhoods, finite) $\mathcal{E}_t = \mathcal{E}_{\mathcal{N}_{t \bmod \mathrm{K}}}$

The intersection of the QL stable sets is the largest invariant set.
Hence, *target is reached if it is the unique state in the intersection*, namely:

$$\mathrm{supp}(\rho) = \bigcap_k \mathcal{H}_{\mathcal{N}_k} := \mathcal{H}_0$$

Proof Sketch - 2 & Remarks

- **ONLY IF part (less intuitive):**

$$\mathcal{E}_{\mathcal{N}_k}(\rho) = \rho \quad \Longrightarrow \quad \mathcal{E}_{\mathcal{N}_k}(\rho_{\mathcal{N}_k} \otimes \tau) = \rho_{\mathcal{N}_k} \otimes \tau$$

$$\rho_{\mathcal{N}_k} = \sum_j q_j |\psi_j\rangle\langle\psi_j|, \quad q_j > 0 \quad \Longrightarrow \quad M_{\mathcal{N}_k}|\psi_j\rangle = \lambda_k |\psi_j\rangle, \forall j$$

Then all states within the local supports must be invariant.

- Cyclic evolution not needed - extensions to randomized evolutions possible:

Corollary: Assume ρ is QLS. Consider an evolution which picks at random one (or more) of the $\{\mathcal{E}_{\mathcal{N}_k}\}$ at each time (independently).
Then ρ is asymptotically prepared if there exists $T, \delta > 0$ such that the probability of picking all of the $\mathcal{E}_{\mathcal{N}_k}$ in each interval $[t, t+T]$ is greater than δ.

➡ **Suitable for unsupervised implementation:** highly robust w.r.t. to sequence ordering, probability distribution, as well as initial state.

QLS, Or Not? Physical Interpretation

- **Equivalent characterization:** $\rho = |\psi\rangle\langle\psi|$ is QLS **if and only if** it is the unique, frustration-free ground state of a QL Hamiltonian, that is:

- There exists a QL Hamiltonian for which $|\psi\rangle$ is the unique ground state and

$$H = \sum_k H_k, \quad H_k = H_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$$

such that $\langle\psi|H_k|\psi\rangle = \min \sigma(H_k), \quad \forall k.$

Proof: It suffices to choose $H_k = \Pi^{\perp}_{\mathcal{N}_k} \otimes I_{\bar{\mathcal{N}}_k}$, $\Pi^{\perp}_{\mathcal{N}_k}$ projects on $\mathrm{supp}(\rho_{\mathcal{N}_k})^{\perp}$.

- Interesting connection to physically-relevant cases, and previous work by Verstraete, Perez-Garcia, Cirac, Wolf, B. Kraus, Zoller and co-workers.
- **Differences:**
  In their setting, the proper locality notion is induced by the target state itself.
  In our setting, *the locality is fixed a priori. We also prove necessity of the condition.*

Is Dissipation Enough?

- **Which states are DQLS? Using our test, it turns out that...**
  - *GHZ states (maximally entangled) and W states are never DQLS* *(unless we have neighborhoods that cover the network - star/complete graphs)*;
  - *Any graph state is DQLS* with respect to **the locality induced by the graph;** To each node is assigned a neighborhood, which contain all the nodes connected by edges.
  - *Generic (injective) MPS/PEPS are QLS* for **some locality definition**... neighborhood size may be big! [see work by Peres-Garcia, Wolf, Cirac and co-workers]
  - Some **Dicke states** *that are not graph* can be stabilized! E.g. on linear graph:

$$\frac{1}{\sqrt{6}}(|1100\rangle + |1010\rangle + |0110\rangle + |0101\rangle + |0011\rangle + |1001\rangle)$$

Good, but not great. Important states are left out. Can we do better?

YES, with a two-step protocol, and conditional stabilization.

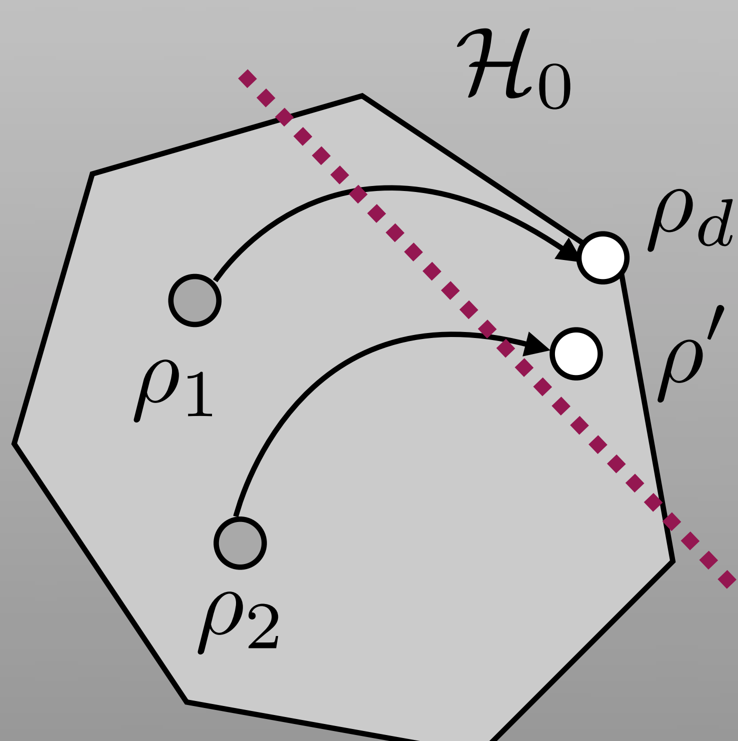
Conditional Preparation: Some Intuition

QLS Problem: unfeasible global stabilization task because I can only prepare (nec. cond.):

$$\mathcal{H}_0 := \bigcap_k \mathcal{H}_{\mathcal{N}_k}$$

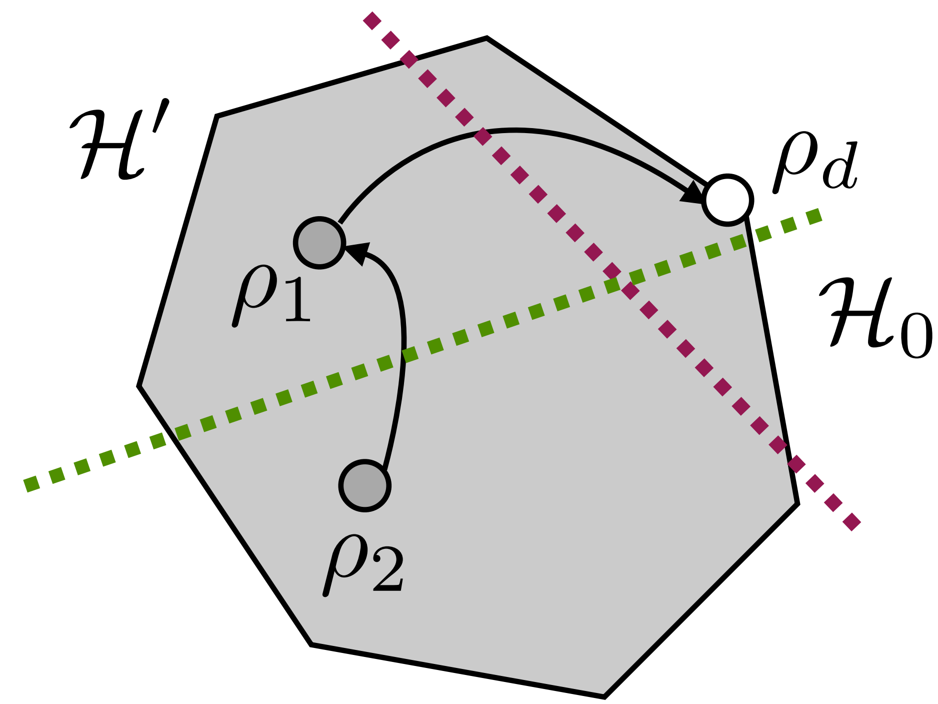
The necessity follows from:

$$\forall\, t \geq 0 \quad \mathcal{E}_t(\rho) = \rho$$



`If we relax this assumptions,`
`we can obtain scalable protocols!`

$$\forall\, t \geq T > 0 \quad \mathcal{E}_t(\rho) = \rho$$



First I prepare a subspace that
(1) is invariant for the QL sequence;
(2) is attracted directly to ρ_d
Problem: finding such $\mathcal{H}'$!

Conditional Preparation: Definition & Result

- **Definition:** A state $\rho = |\psi\rangle\langle\psi|$ is **Quasi-Local Stabilizable (QLS) conditional to** $\mathcal{H}'$ if there exist a **sequence of QL channels** $\{\mathcal{E}_t\}_{t\geq 0}$ such that

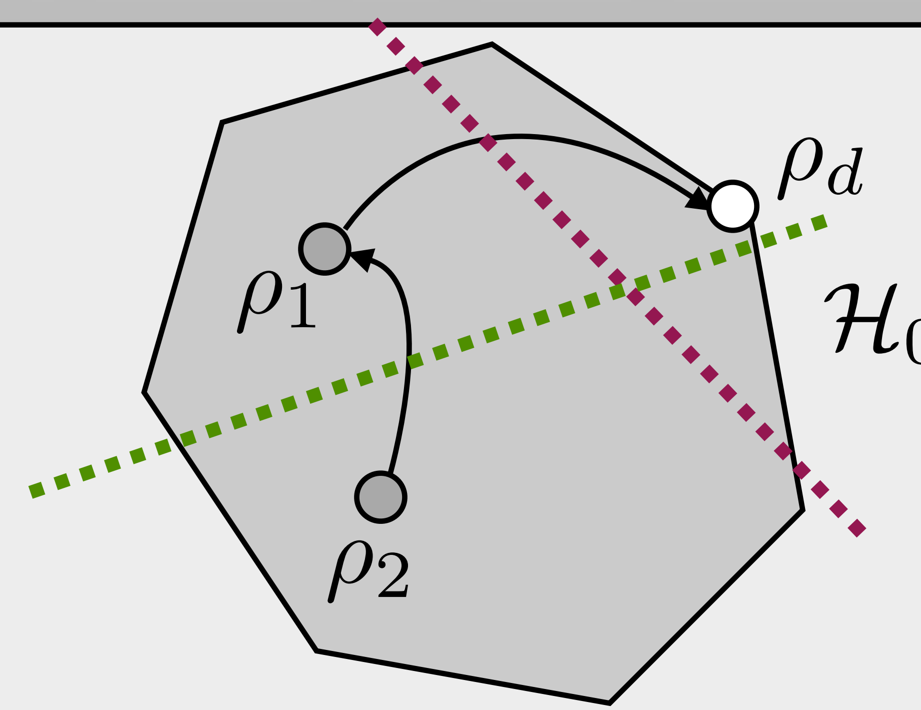
$$\forall\, t \geq 0 \quad \mathcal{E}_t(\rho) = \rho \qquad \lim_{t\to\infty} \|\rho_t - \rho\| = 0$$

for every ρ_0 with support on $\mathcal{H}'$.

With some additional hypothesis, the search for the subspace can be automated.

- **Lemma:** It is not restrictive to take $\mathcal{H}'$ invariant.

- **Theorem:** If $\mathcal{H}'$
  (1) contains $|\Psi\rangle$;
  (2) is orthogonal to $\mathcal{H}_0 \ominus \{|\Psi\rangle\}$;
  (3) is invariant for $\{\mathcal{E}_t\}_{t\geq 0}$ that stabilizes $\mathcal{H}_0$;
  Then $\rho = |\psi\rangle\langle\psi|$ is QLS conditional to $\mathcal{H}'$.



A case study: GHZ States

- **GHZ states are never QLS for non trivial topology:**

$$\rho_{\text{GHZ}} = |\Psi\rangle\langle\Psi|, \quad |\Psi\rangle \equiv |\Psi_{\text{GHZ}}\rangle = (|000\ldots0\rangle + |111\ldots1\rangle)/\sqrt{2}.$$

By symmetry, $\mathcal{H}_0$ **must contain** $|000\ldots0\rangle, |111\ldots1\rangle$ **.**

Hence the following orthogonal states **must remain** stable for the QL dynamics.

$$\begin{aligned} |\Psi_{\text{GHZ}^+}\rangle &= (|000\ldots0\rangle + |111\ldots1\rangle)/\sqrt{2}; \\ |\Psi_{\text{GHZ}^-}\rangle &= (|000\ldots0\rangle - |111\ldots1\rangle)/\sqrt{2}; \end{aligned}$$

We need to "select" the right one How?

$$\sigma_x^{\otimes n}|\Psi_{\text{GHZ}^+}\rangle = |\Psi_{\text{GHZ}^+}\rangle \quad \sigma_x^{\otimes n}|\Psi_{\text{GHZ}^-}\rangle = -|\Psi_{\text{GHZ}^-}\rangle$$

- **Trick:** First prepare the system in the +1-eigenspace of $\sigma_x^{\otimes n}$ (e.g. $|+\rangle^{\otimes n}$). Then we show there exists a QL $\{\mathcal{E}_t\}_{t\geq 0}$ that prepares $\mathcal{H}_0$ leaving the **eigenspace invariant.**

- By our Theorem, ρ_{GHZ} **is Conditionally QLS!** (scalable on the linear graph)

Can I use QL dynamics
to globally symmetrize the network?

Quantum Consensus & Group Symmetrization

[Mazzarella, Sarlette, T, arxiv:1303.4077 & 1311.3364 & MTNS 2014]

Why Quantum Symmetrization?

- **Single System - Ubiquitous Concept:**
  - Symmetrize the state with respect to a unitary group is connected to **thermalization**;
  - Symmetric dynamics allows for **preserved quantities, preserved information, protected codes** [Zanardi, Knill, LaFlamme, Viola & coworkers, Wolf];
  - Symmetrize dynamics (e.g. via impulsive, **quantum dynamical decoupling** techniques) allows for engineering desired Hamiltonians and removing “noise” components [Viola-Knill-Lloyd and before that NMR];

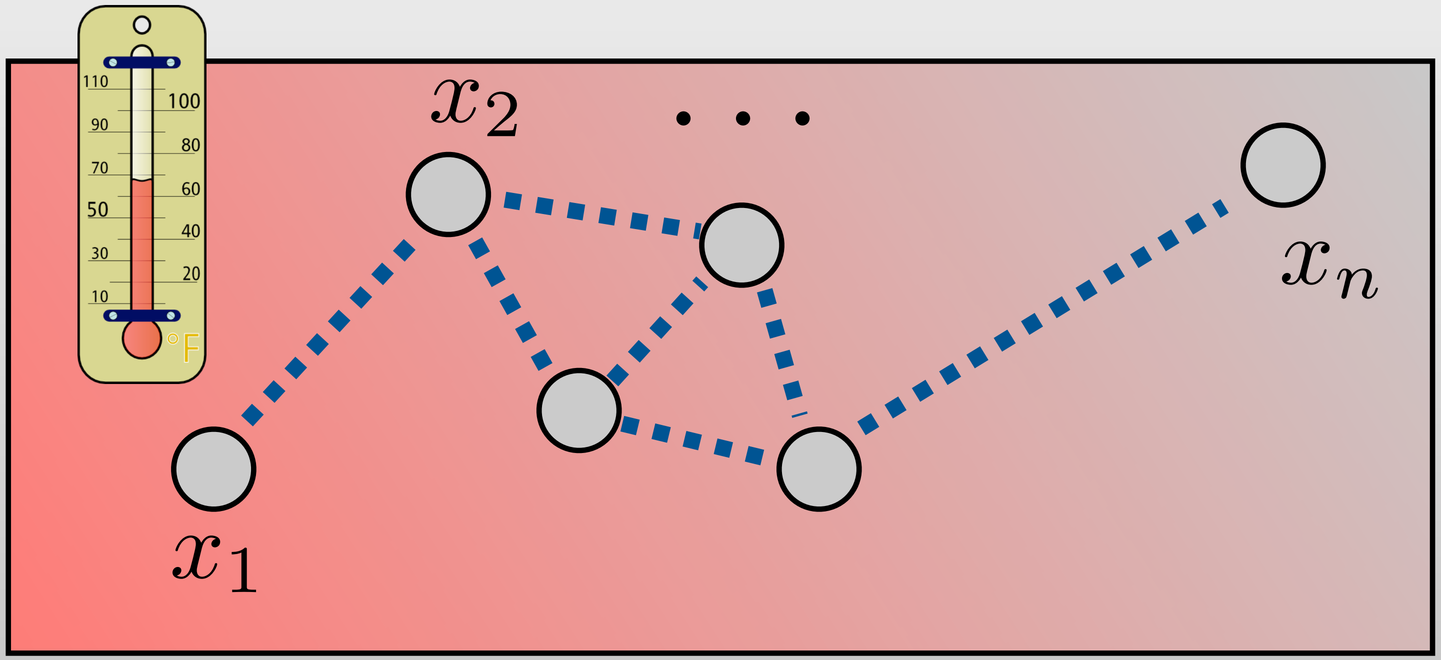
➡ **“Network” of interacting systems?**

- *Task 1:* Have “uniformly distributed” classical or quantum information on the nodes (subsystems);
- *Task 2*: Mixing or “Thermalization” of a large sample.

More motivation coming later... **Tool for control!**

Motivating Classical Application

- **Consider n sensors (agents) probing the temperature of a room:**



- Each agent records a (different) value, x_j , due to errors and temp. gradient;
- **Then it broadcasts the value on a *limited* communication network;**
- **The task:** have the sensor **agree** on the initial mean value (*average consensus*);

$$\forall, j, k \;\; x_j(\infty) = x_k(\infty) = \bar{x}(0) := \sum_j x_j(0)$$

How to (asymptotically) reach it?

- Can the protocol be made robust w.r.t. size, sequence, initial values....?

Rewriting the Gossip Algorithm

- **A simple Gossip algorithm:**
  - At each t, one communication link (graph) edge is selected, say **(j,k)**;
  - The two agents share their values and compute the (weighted) mean:

$$\begin{aligned} x_j(t+1) &= x_j(t) + \alpha(t)(x_k(t) - x_j(t)) \\ x_k(t+1) &= x_k(t) + \alpha(t)(x_j(t) - x_k(t)) \end{aligned}$$

  - Under weak assumptions on how they are selected, it converges.
- Let us rewrite the interaction as **the convex sum of two permutations:**

$$(x_j(t+1),\ x_k(t+1)) = (1\text{-}\alpha(t))\ (x_j(t),\ x_k(t))\ +\ \alpha(t)\ (x_k(t),\ x_j(t))$$

or, including all readings in one vector: $x(t) = (x_1(t), \ldots, x_n(t))$

$$x(t+1) = ((1-\alpha(t))I + \alpha(t)P_{jk})\, x(t)$$

Distributed,
Robust,
Unsupervised,
Scalable

Rewriting Classical Consensus

- Thus, the evolution up to time t can be always written as

$$x(t) = \left(\sum_{\pi \in \mathfrak{P}} p_\pi(t) P_\pi \right) x(0)$$

$p_\pi(t)$ **:** non-negative weights, sum up to one.

π **:** runs over the finite group of permutation of n elements;

- **Consensus is obtained if the state is permutation invariant, and the average is preserved;**
- In order for this to happen asymptotically, we need:

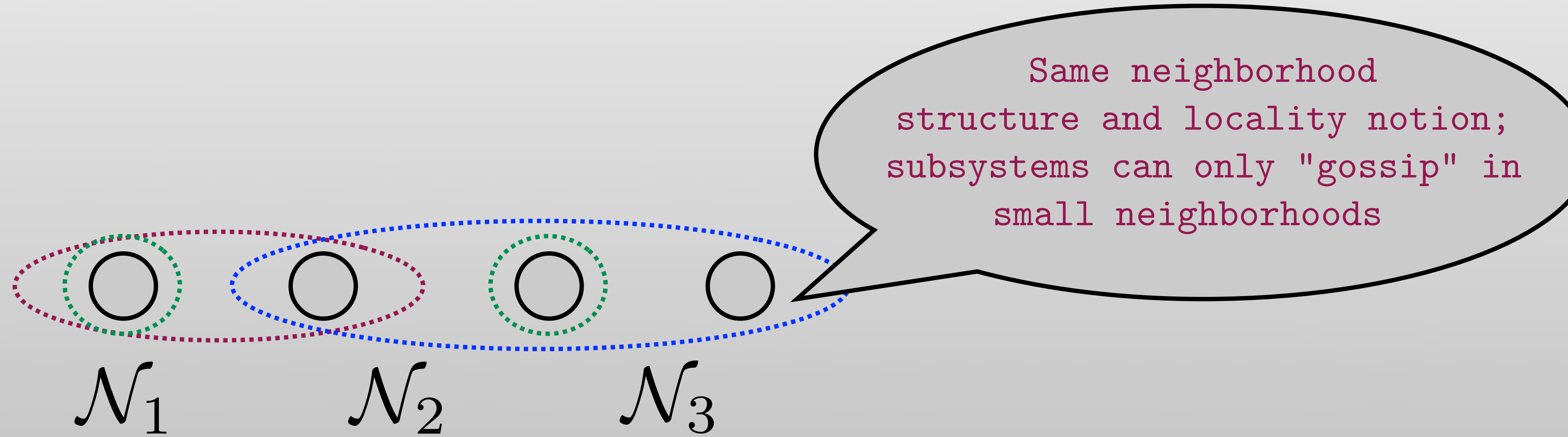
$$x(\infty) = \frac{1}{n!} \sum_{\pi \in \mathfrak{P}} P_\pi x(0)$$

`Projection on the group-symmetric configuration!`

- **Under very weak assumptions on the selection rule, it converges!**

Quantum Consensus

- We can then use the same ideas to **symmetrize a quantum network.**
- Quantum Network of *n* subsystems with dynamics subject to QL constraints.



- Consider subsystem permutation operators defined on a factorized basis (and extended by linearity) by:

$$U_\pi^\dagger (X_1 \otimes \ldots \otimes X_m) U_\pi = X_{\pi(1)} \otimes \ldots \otimes X_{\pi(m)}$$

- We wanted to generate a **uniform convex sum of subsystem permutations. Now unital CPTP map of the form:**

$$\bar{\mathcal{E}}(\rho) = \frac{1}{n!} \sum_{\pi \in \mathfrak{P}} U_\pi^\dagger \rho U_\pi$$

[projection onto symmetric states, centralizer]

Quantum Gossip Algorithm

- **Consider maps of the form:**

$$\mathcal{E}_t(\rho) = \alpha(t)\rho + (1-\alpha(t))U_{j,k}(t)\rho U^\dagger_{j,k}(t),$$

  where $\{U_{j,k}\}$ are QL "swap" operators, and $\alpha(t) \in [0,1]$.

- **Same form of the classical case!**

- **Proposition:** The fixed points are the permutation invariant states if and only if the $\{U_{j,k}\}$ generate the full permutation group.

- **Idea:** The dynamics is unital, i.e. preserves the identity. Fixed point for unital semigroups are the commutant of the algebra generated by the $\{U_k\}$.

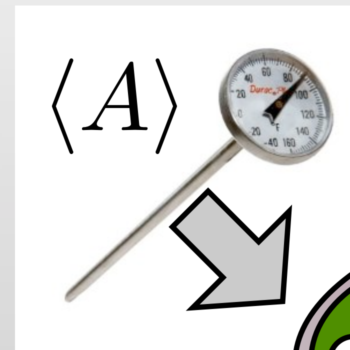
- **Result: Under the same weak assumptions of the classical case, a switching dynamics with QL operators converges!**
  [More fundamental reasons... see the poster!]

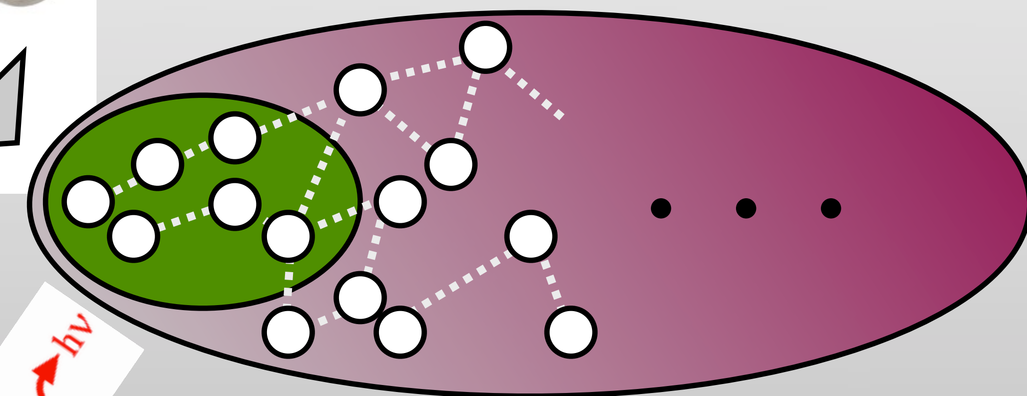
Applications of Quantum Consensus

- **Mixing for Partial Sampling:**

  Average observable estimation

$$A = \frac{1}{n} \sum_j A^{(j)}$$





- **Factorized State Preparation with Localized Control:**

  -> Purify a locally accessible sample;

  -> Use gossip to mix;

  -> Iterate;

Gossip algorithm on the whole network; Complete control & measurements on a portion

- **Size Estimation of a Quantum Network:**

  -> Prepare the network in a pure state (as above) $|\phi\rangle^{\otimes n}$;

  -> Prepare the sample in an **orthogonal state** $|\psi\rangle^{\otimes m}$**;**

  -> Use gossip to mix;

  -> Estimate the average expectation of A as above, with

$$A = \frac{1}{n} \sum_j |\psi\rangle\langle\psi|^{(j)}$$

The Message...

- Using system-theoretic methods, we derived a framework for
  - **Stability analysis of Markov dynamics on multipartite systems;**
  - **Tests for checking pure QLS states;**
  - **Constructive results for pure entangled state preparation under locality constraints; Robust methods;**
  - **Scalable protocols for conditional preparation of GHZ states;**
  - **Symmetrization methods;**

➡**Open problems:**
Better *classification* of QLS states;
Speed of convergence (when the system size grows - scalability);
Robustness w.r.t. control errors;

➡**Next:**
Mixed states preparation (almost there!); New applications of Consensus; Stronger Consensus; Thermalization; Non-Markovian Models; ...