Measuring and Allocating Systemic Risk

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Questions

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3. How to manage systemic risk?

Short answers

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   - Satisfy the clone property

3. Systemic risk can be managed by
   - Setting systemic risk limits (capital requirements)
   - Imposing systemic risk charges (Pigouvian tax)
   - Cap and trade system for systemic risk

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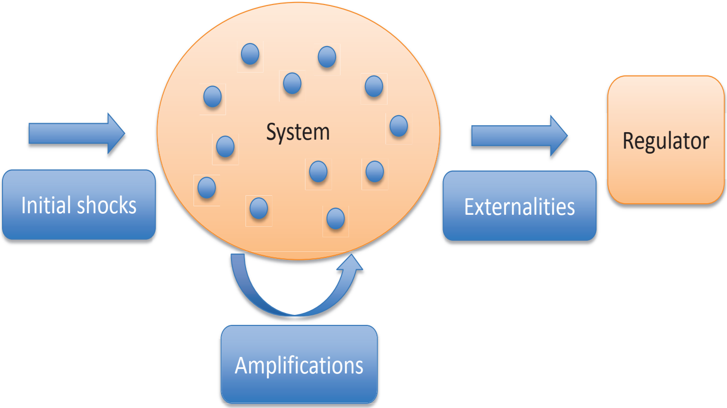
4. Managing systemic risk

Underlying Stochastic Model



Two-Step Reduced Form Model

The sample space: $\Omega = \{\omega_1, \ldots, \omega_d\}$

Random variables: $L = \{X : \Omega \to \mathbb{R}\} = \mathbb{R}^d$

Real GDP: $Y = y(F)$

- $F_1, \ldots, F_n \in L$ random macro variables
- $y : \mathbb{R}^n \to \mathbb{R}$

Initial net worth of institution *i*: $x_i > 0$

(e.g. market value of assets minus book value of liabilities)

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Step 1 Economic shocks

$$x_i \in \mathbb{R} \longrightarrow X_i = z_i(F, W_i) \in L$$

$W_i \in L$ idiosyncratic shock, $z_i : \mathbb{R}^{n+1} \to \mathbb{R}$

Step 2 Amplification mechanisms

- Direct spillovers
  - Contractual connections
- Indirect spillovers
  - Fire sale externalities
  - Dry up of funding liquidity

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$$X_i \longrightarrow V_i = a_i(F, X, c, l) \in L$$

- c_{ij}, $i \neq j$, interconnectedness parameters
- $l_1, \ldots, l_I$ liquidity mismatch indexes
- $a_i : \mathbb{R}^{n+I+I^2} \to \mathbb{R}$

1. Measuring Total Systemic Risk

Externalities Caused by the Financial Sector

$$E_i = -\alpha_i V_i^- + \beta_i (V_i - v_i)^+$$

where

- $V_i \in L$ net worth at the end of the measuring period
- $\alpha_i,\ \beta_i, v_i \geq 0$ constants
- $\alpha_i V_i^-$ cost of bailout or restructuring
  (α_i can depend on firm i's complexity)
- $\beta_i (V_i - v_i)^+$ more tax revenues than normal

Aggregate externality: $E = \sum_i E_i$

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Denote

$$\operatorname{dom} U := \{X \in L : U(X) > -\infty\}$$

and assume

- $\operatorname{int} \operatorname{dom} U \neq \emptyset$
- Strict monotonicity on $\operatorname{int} \operatorname{dom} U$
- Differentiability on $\operatorname{int} \operatorname{dom} U$

Natural Example: CRRA expected utility

$U(X) = \mathbb{E}_{\mathbb{P}}[u(X)]$

for a probability measure with full support $\mathbb{P}$ and

$$u(x) = \begin{cases} \frac{1}{1-\gamma} x^{1-\gamma} & \text{if } x > 0 \\ -\infty & \text{if } x \leq 0 \end{cases} \qquad \text{for some } \gamma > 1$$

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$\operatorname{dom} U = \operatorname{int} \operatorname{dom} U = \mathbb{R}^d_{++}$ and $\nabla U(X) \cdot Z = \mathbb{E}_{\mathbb{P}}[u'(X) Z]$

1. Measuring Total Systemic Risk

SystRisk

Fix a tolerance level $e \leq 0$ and assume $U(Y + e) \in \mathbb{R}$

$$\rho(E) := \inf\{m \in \mathbb{R} : U(Y + E + m) = U(Y + e)\}$$

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$\rho : \mathbb{R}^d \to \mathbb{R}$ satisfies

Normalization at e $\quad \rho(e) = 0$

Monotonicity $\quad \rho(E) \geq \rho(E')$ if $E \leq E'$

Translation property $\quad \rho(E-m) = \rho(E) + m$ for $m \in \mathbb{R}$

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Insurance Valuation Principle as opposed to

Risk-Neutral Valuation (no hedging!)

Proposition

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Corollary (superlinear scaling)

If U is strictly concave on $\operatorname{int}\operatorname{dom} U$ and E is non-constant, then

$$\rho(\lambda E) > \lambda\rho(E) \quad \text{for } \lambda > 1$$

1. Measuring Total Systemic Risk

Different from e.g.

- VaR
- Expeced Shortfall
- Adrian and Brunnermeier (2009)
- Acharya, Pedersen, Philippon and Richardson (2010)
- Acharya, Engle and Richardson (2012)
- Chen, Iyengar and Moallemi (2013)

2. Systemic Risk Allocation

Theorem Assume

$$\text{(I)} \quad Y + E + \rho(E) \in \operatorname{int} \operatorname{dom} U.$$

Then

$$\lim_{\varepsilon \to 0} \frac{\rho(E + \varepsilon E') - \rho(E)}{\varepsilon} = \mathbb{E}_{\mathbb{Q}^E}[-E']$$

for

$$\mathbb{Q}^E := \frac{\nabla U(Y + E + \rho(E))}{\|\nabla U(Y + E + \rho(E))\|_1}$$

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Shadow measure for systemic risk

For $U = \text{CRRA}$,

$$\mathbb{Q}^E := \frac{(Y + E + \rho(E))^{-\gamma}}{\mathbb{E}_{\mathbb{P}}(Y + E + \rho(E))^{-\gamma}}$$

2. Systemic Risk Allocation

1. Marginal contributions

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(Euler allocation)

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satisfies

(AD) Additivity

If firms i and j merge so that $E_{i+j} = E_i + E_j$ and the other E_k stay the same, then $MC_{i+j} = MC_i + MC_j$ and the other MC_k stay the same.

MC_i is the marginal version of the

2. With-without allocation

$$WW_i := \rho(E) - \rho(E - E_i) \quad \text{(Merton and Perold, 1993)}$$

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2. With-without allocation

$$WW_i := \rho(E) - \rho(E - E_i) \quad \text{(Merton and Perold, 1993)}$$

which satisfies

(CR) Causual responsibility

If the i-th externality changes from E_i to $E_i + \Delta E_i$, then $\Delta WW_i = \rho(E + \Delta E_i) - \rho(E)$

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where

- $s_i \geq 0$ are exogenous size parameters,
  e.g. corporate taxes paid last year
- $\mu \geq 0$ is chosen so that one has

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SMC_i satisfy (FA), (AD) and (CR) approximately

3. Mergers and Spinoffs

Let's merge firms $i_1, \ldots i_m$.

- Assume initial shocks and size parameters add up:

$$\hat{X} = \sum_{j=1}^{m} X_{i_j}, \quad \hat{s} = \sum_{j=1}^{m} s_{i_j}$$

- SMC_i are additive in E_i

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But

- Amplification effects are non-linear: $\hat{V} \neq \sum_{j=1}^{m} V_{i_j}$
- Externalities $E_i = -\alpha_i V_i^- + \beta_i (V_i - v_i)^+$ are non-linear in V_i

Clone Property

If $i_1, \ldots, i_m$ are clones, then

- $\hat{X} = \sum_{j=1}^{m} X_{i_j} = mX_{i_1}$
- $\hat{V} = \sum_{j=1}^{m} V_{i_j} = mV_{i_1}$
- $\hat{E} = \sum_{j=1}^{m} E_{i_j} = mE_{i_1}$

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- $\hat{E} = \sum_{j=1}^{m} E_{i_j} = mE_{i_1}$

Consequently,

- $S\hat{M}C = \sum_{j=1}^{m} SMC_{i_j} = mSMC_{i_1}$
- Total SystRisk and the other SMC_k do not change

Reducing Systemic Risk Through Spinoffs

Split a large financial institution into m business units.

1. Reduces negative feed back effects (firewalls)

   "$\hat{V} \leq \sum_{j=1}^{m} V_{i_j}$"

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2. Reduces complexity: $\hat{\alpha} \leq \alpha_{i_j}$

Consequently,

- Total systemic risk is reduced
- The contribution of the group $i_1, \ldots, i_m$ is reduced

Cases Where Spinoffs Increase Systemic Risk

If spinoffs do not reduce complexity, then

- Amplifcation effects are not reduced
- Bailout costs are not reduced: $\hat{\alpha} = \alpha_{i_1} = \cdots = \alpha_{i_m}$

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Consequently,

- $\hat{E} \geq \sum_{j=1}^{m} E_{i_j}$

  (spinoffs reduce diversification/netting effects)
- Systemic risk increases

4. Managing Systemic Risk: A Discussion

A) Setting risk limits (capital requirements)

B) Systemic risk charges (Pigouvian taxes)

C) Cap and trade

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- Makes the approach independent of behavioral assumptions
- Response has to monitored if regulation is implemented
- Tolerance parameter can be adjusted to find optimal level of regulation
- Our model does not say which of A)–C) is best

  compare to Weitzman (1974): Prices vs. quantities

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A) Setting risk limits $SMC_i \leq 0$

- Individual risk limits guarantee $\rho(E) \leq 0$
- If $\rho(E) > 0$
  - $-\nabla\rho(E)$ gives direction of most efficient risk reduction
  - Can be translated into systemic capital requirements

4. Managing Systemic Risk: A Discussion

B) Systemic risk charges $\tau_i = SMC_i^+$

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- Allows for the pooling of risk capital
- Generally, risk charges are non-linear in the externalities

C) Cap and trade

Risk permits issued in the amount P

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Risk permits issued in the amount P

$$\text{Requirement:} \quad SMC_i - P_i \leq 0$$

- Can be traded
- Gives a market price of systemic risk
- Firms with low abatement costs will reduce most of the systemic risk

Conclusion

- General approach to measuring and allocating systemic risk
- Depends on the underlying model of shocks and amplifications
- Needs consideration:

  How does regulation affect the system?