Transmission capacity allocation in zonal electricity markets

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Outline

1. Introduction
2. Day-ahead zonal electricity market models
3. Policy analysis using 4-node, 3-zone network
4. Cutting-plane algorithms for robust day-ahead zonal electricity markets
5. Simulation results for the Central Western European network
6. Conclusions

Introduction

Zonal electricity markets

- European electricity market organized as a zonal market, EC 714/2009
- Two types of export/import limits:
  - Limits on the bilateral exchange between neighbouring zones, Available-Transfer-Capacity Market Coupling (**ATCMC**)
  - Limits on the net position configuration of zones, Flow-Based Market Coupling (**FBMC**)
- Both methodologies should be **N-1 robust** (*Critical Branches/Critical Outages*), Amprion *et al.* (2017)
- FBMC used to clear day-ahead electricity market at the Central Western European system since May 2015
- Other markets might implement FBMC in the near future (e.g. Nord Pool, Energinet *et al.* (2017))

Flow-Based Market Coupling (FBMC)

- Preferred methodology for electricity market operations of the EC, EU 2015/1222: "*... a method that takes into account that electricity can* **flow via different paths** *and optimizes the available capacity in* **highly interdependent grids** *...*"
- Increases in day-ahead market welfare of **95M€/year** with respect to ATCMC, Amprion *et al.* (2013)
- Congestion management and balancing costs not included in studies. They amounted to **945M€** in 2015, ENTSO-E (2015).
- Questions:
  - Do FBMC or ATCMC correctly account for physical flows? Why do they/do they not?
  - Does FBMC significantly improve the overall welfare of the market with respect to ATCMC?

Literature

- Jensen *et al.* (2017), and many references therein, study ATCMC and conclude that its performance is significantly **worse than that of a nodal market**
- Waniek *et al.* (2009), Waniek *et al.* (2010) study the accuracy of the approximation of flows in FBMC at cross-border lines
- Marien *et al.* (2013) study how **discretionary aggregation parameters** (for export/import limits) affect the outcome of FBMC
- Van den Bergh *et al.* (2015) summarize the concepts and methodology used for FBMC at the Central Western European system
- Dierstein (2017) analyzes the impacts of **discretionary aggregation parameters** on welfare, exchanges, prices and counter-trading costs

Contributions

1. We propose a new framework for modelling ATCMC and FBMC in which we derive export/import limitations directly from the physics of the real network.

   (the proposed models **do not depend** on discretionary aggregation parameters)

2. We present cutting-plane algorithms to systematically account for the N-1 security criterion on day-ahead markets.

3. We perform numerical simulations using an industrial-scale instance of the Central Western European system considering 100 years of operating conditions.

   - Vast similarities between ATCMC and FBMC in all aspects.

   - Zonal market designs fail at allocating transmission capacity and are outperformed by a nodal market.

Day-ahead electricity market models

Assumptions

1. All energy trades take place at the day-ahead auction
2. Bids are price-quantity pairs, associated with a node/zone
3. Participants bid truthfully
4. System operator knows:
   - Topology of the network
   - Susceptance and thermal limits of lines
   - Installed production capacity at each node
5. Consumers have an infinite valuation (only for simplicity)

Nodal electricity market

$$\min_{\text{bids, flows}} \quad \text{production cost}$$

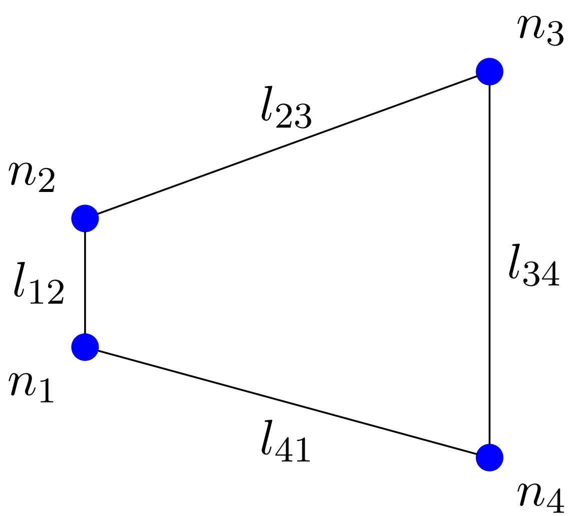
$$\text{s.t.} \quad \text{fractional bids}$$

net production $=$

outgoing flows, at each node

line thermal limits

power-angle constraints



Nodal electricity market

$$\min_{v,f,\theta} \quad \sum_{g \in G} P_g Q_g v_g$$

$$\text{s.t.} \quad 0 \leq v_g \leq 1 \quad \forall g \in G$$

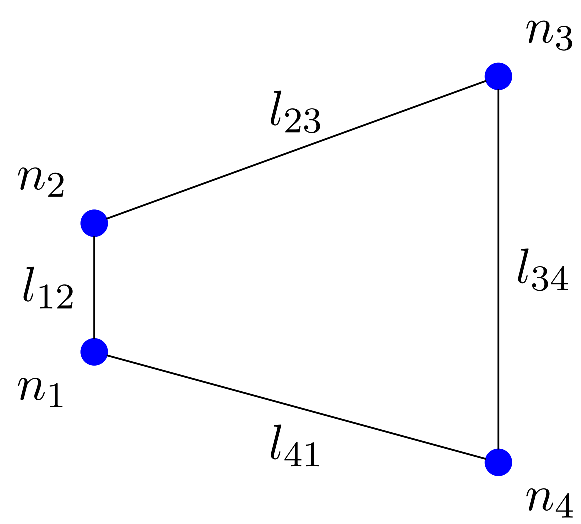
$$\sum_{g \in G(n)} Q_g v_g - Q_n = \sum_{l \in L(n,\cdot)} f_l - \sum_{l \in L(\cdot,n)} f_l \quad \forall n \in N \quad [\rho_n]$$

$$-F_l \leq f_l \leq F_l \quad \forall l \in L$$

$$f_l = B_l \left(\theta_{m(l)} - \theta_{n(l)}\right) \quad \forall l \in L$$

P, Q: price and quantity

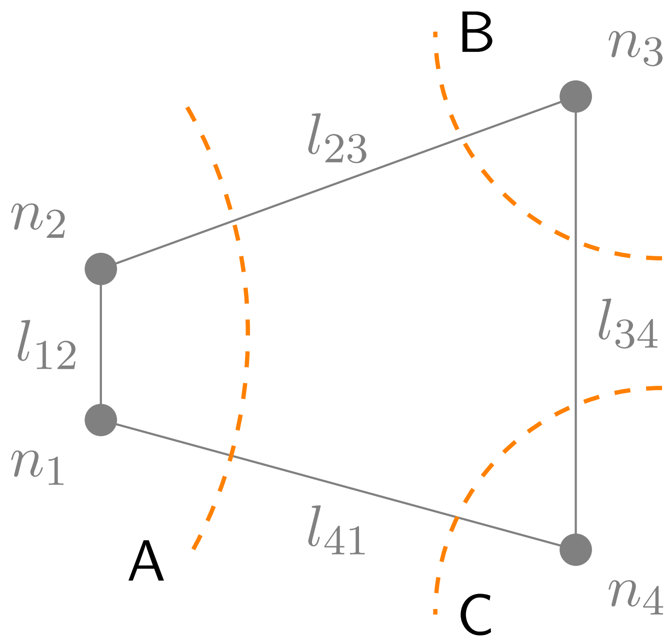
F_l, B_l: capacity and susceptance line l



$$G = \{1, 2, 3, 4\},$$
$$G(n_1) = \{1\}, \ldots$$
$$N = \{n_1, n_2, n_3, n_4\}$$

$$L = \{l_{12}, l_{23}, l_{34}, l_{41}\},$$
$$L(n_1, n_2) = \{l_{12}\}, \ldots$$

Zonal network organization

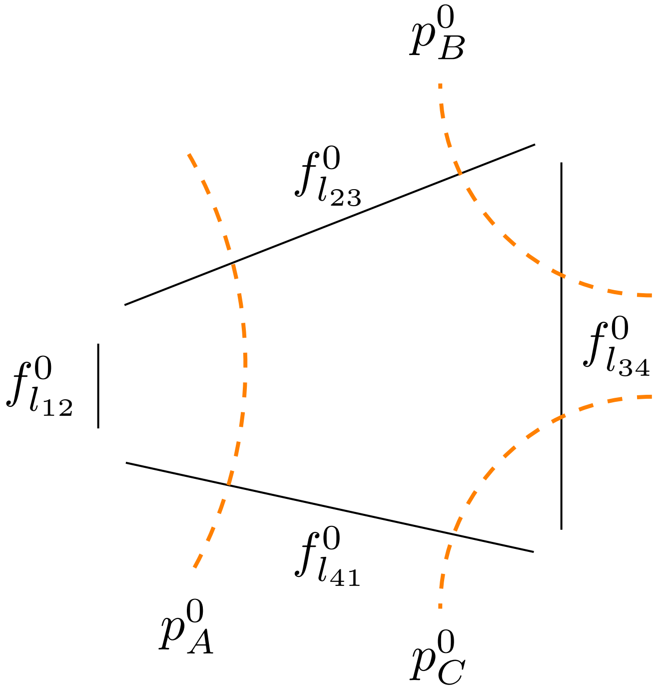


$$G = \{1, 2, 3, 4\},\ G(A) = \{1, 2\}, \dots$$

$$N = \{n_1, n_2, n_3, n_4\},\ N(A) = \{n_1, n_2\}, \dots$$

Flow-Based Market Coupling with Approximation (FBMC-A)

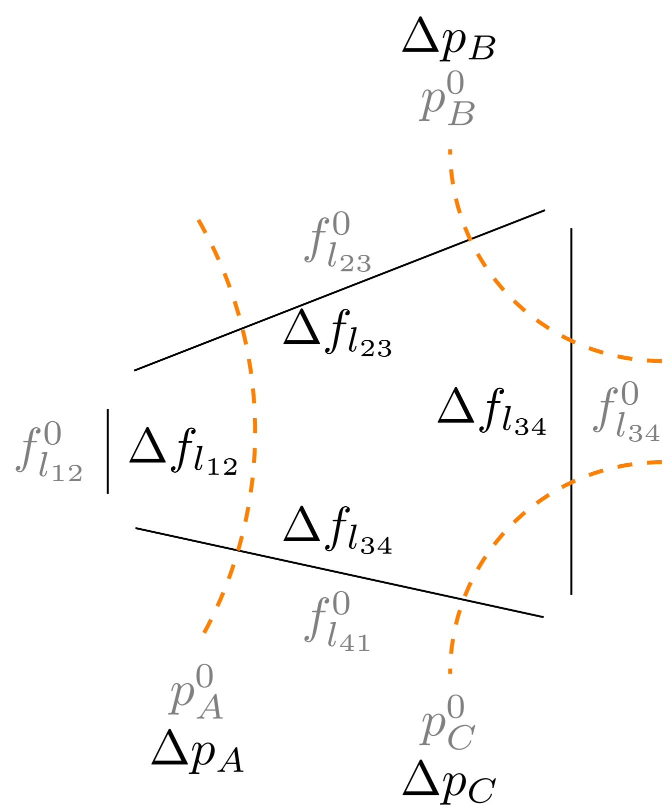
1. Select a base case (p^0, f^0) (net positions, flows on branches)



Flow-Based Market Coupling with Approximation (FBMC-A)

1. Select a base case (p^0, f^0) (net positions, flows on branches)
2. Compute zone-to-line Power-Transfer-Distribution-Factors, $PTDF_{l,z}$, so that

$$\Delta f_l \approx \sum_{z \in Z} PTDF_{l,z} \Delta p_z$$



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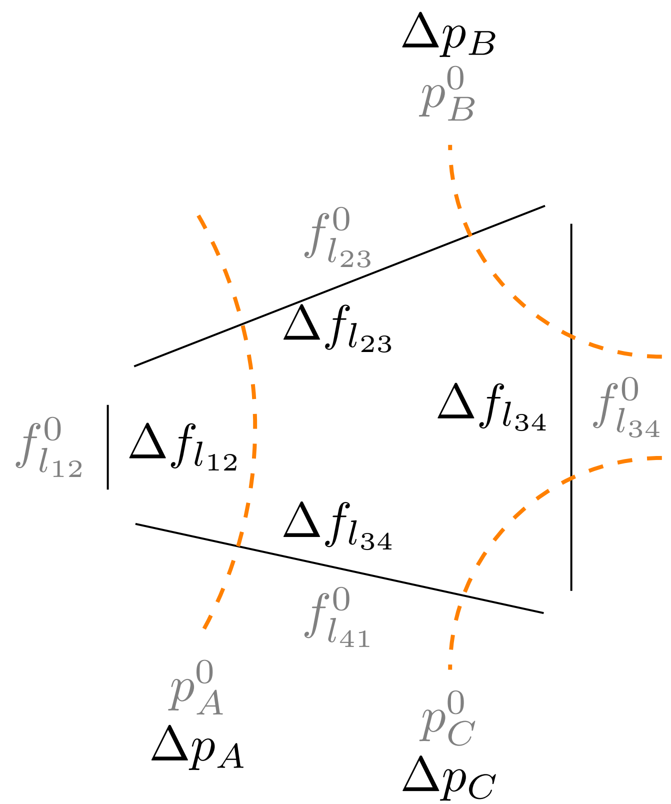
$$\Delta f_l \approx \sum_{z \in Z} PTDF_{l,z} \Delta p_z$$

3. Define **flow-based domain**:

$$\mathcal{P}^{FB-A} := \left\{ p \in \mathbb{R}^{|Z|} \middle| \sum_{z \in Z} p_z = 0, \right.$$

$$\left. -F_l \leq \sum_{z \in Z} PTDF_{l,z}(p_z - p_z^0) + f_l^0 \leq F_l \quad \forall l \in L \right\}$$



Flow-Based Market Coupling with Approximation (FBMC-A)

4. Clear day-ahead market by solving:

$$\min_{v,p} \sum_{g \in G} P_g Q_g v_g$$

$$\text{s.t.} \quad 0 \leq v_g \leq 1 \quad \forall g \in G$$

$$\sum_{g \in G(z)} Q_g v_g - \sum_{n \in N(z)} Q_n = p_z \quad \forall z \in Z \quad [\rho_z]$$

$$\sum_{z \in Z} p_z = 0$$

$$-F_l \leq \sum_{z \in Z} PTDF_{l,z}(p_z - p_z^0) + f_l^0 \leq F_l \quad \forall l \in L$$

- □ Circular definitions: base case (p^0, f^0), market clearing point
- □ Discretionary parameters: zone-to-line PTDF (among others)

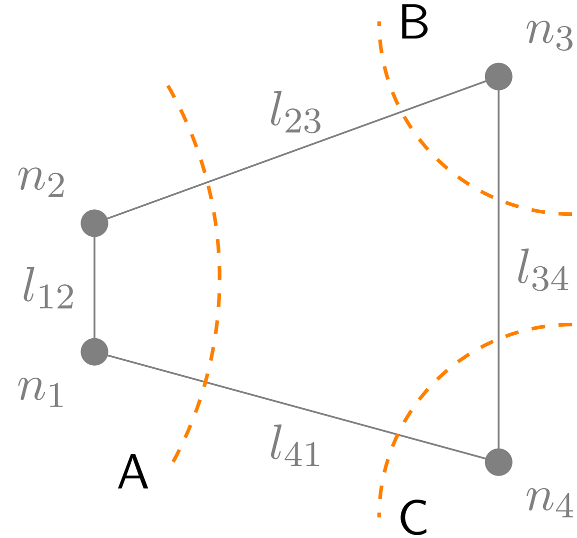
Zonal electricity market

$$\min_{v,p} \quad \sum_{g \in G} P_g Q_g v_g$$

$$\text{s.t.} \quad 0 \leq v_g \leq 1 \quad \forall g \in G$$

$$\sum_{g \in G(z)} Q_g v_g - \sum_{n \in N(z)} Q_n = p_z \quad \forall z \in Z \quad [\rho_z]$$

$$p \in \mathcal{P}$$



$$G = \{1, 2, 3, 4\},$$
$$G(A) = \{1, 2\}, \ldots$$

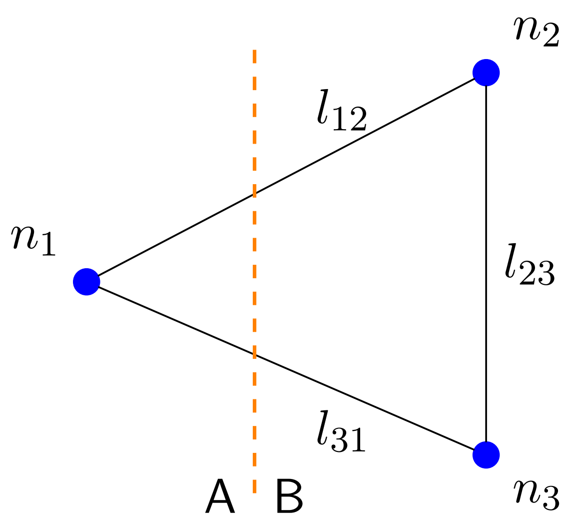
$$N = \{n_1, n_2, n_3, n_4\},$$
$$N(A) = \{n_1, n_2\}, \ldots$$

- $\mathcal{P}$ should include all feasible cross-border trades, EC 714/2009, Annex I, Art. 1.1
- $\mathcal{P}$ should not include configurations that can harm security, EC 714/2009, Annex I, Art. 1.7

Deriving $\mathcal{P}$ directly from physics: an example

Physics:

$$r_1 + r_2 + r_3 = 0$$
$$-100 \leq r_1 \leq 100$$
$$-100 \leq r_2 \leq 100$$
$$-100 \leq r_3 \leq -50$$
$$-25 \leq f_{12} = 1/3\, r_1 - 1/3\, r_2 \leq 25$$



$$G = \{1, 2, 3\}$$
$$Q_1 = 200,\ Q_2 = 200,\ Q_3 = 50$$
$$N = \{n_1, n_2, n_3\}$$
$$L = \{l_{12}, l_{23}, l_{31}\},\ F_{12} = 25$$

100MW demand per node

Zonal net positions:

$$p_A = r_1$$
$$p_B = r_2 + r_3$$

Deriving $\mathcal{P}$ directly from physics: an example

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Zonal net positions:

$$p_A = r_1$$
$$p_B = r_2 + r_3$$

Are these zonal net positions feasible?

$$p_A = 0 \qquad p_B = 0$$
$$p_A = 200 \qquad p_B = -200$$
$$p_A = -100 \qquad p_B = 100$$
$$p_A = 50 \qquad p_B = -50$$

Deriving $\mathcal{P}$ directly from physics: an example

Physics:

$$
\begin{aligned}
r_1 + r_2 + r_3 &= 0 \\
-100 \leq r_1 &\leq 100 \\
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\end{aligned}
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Zonal net positions:

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p_A &= r_1 \\
p_B &= r_2 + r_3
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Are these zonal net positions feasible?

$p_A = 0$	$p_B = 0$	Yes
$p_A = 200$	$p_B = -200$	
$p_A = -100$	$p_B = 100$	
$p_A = 50$	$p_B = -50$	

Deriving $\mathcal{P}$ directly from physics: an example

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Zonal net positions:

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$p_A = 0$	$p_B = 0$	Yes
$p_A = 200$	$p_B = -200$	No
$p_A = -100$	$p_B = 100$	
$p_A = 50$	$p_B = -50$	

Deriving $\mathcal{P}$ directly from physics: an example

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Zonal net positions:

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Zonal net positions:

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Deriving $\mathcal{P}$ directly from physics: an example

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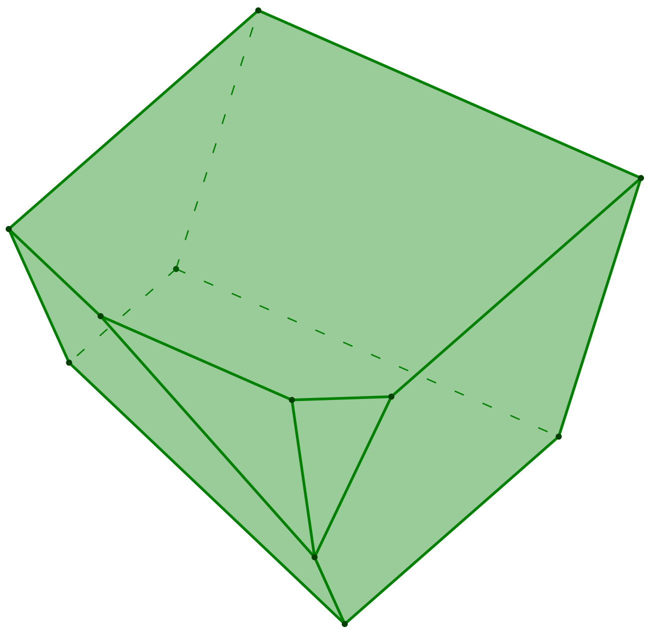
$p_A = 200$	$p_B = -200$	No
$p_A = -100$	$p_B = 100$	No
$p_A = 50$	$p_B = -50$	Yes

True net position feasible set $\mathcal{P}$:

$$p_A + p_B = 0$$
$$-12.5 \leq p_A \leq 87.5$$

Flow-Based Market Coupling with Exact Projection (FBMC-EP)

space of nodal injections $\rightarrow$ space of zonal net positions

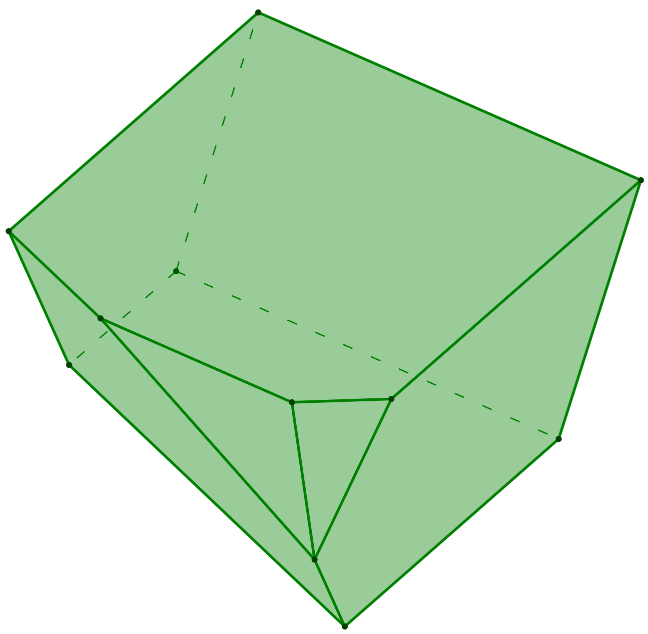


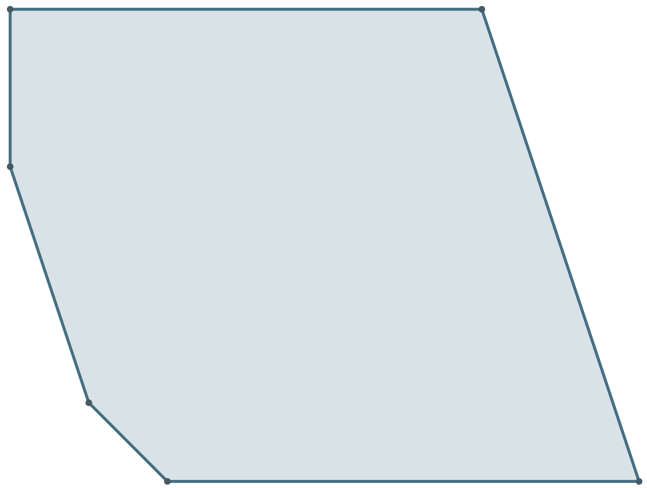
$$\mathcal{R} := \Big\{ r \in \mathbb{R}^{|N|} \Big| \; r \text{ is feasible for the real network} \Big\}$$

4-node, 3-zone network: $p_A = r_1 + r_2,\; p_B = r_3,\; p_C = r_4$

Flow-Based Market Coupling with Exact Projection (FBMC-EP)

space of nodal injections $\rightarrow$ space of zonal net positions





$$\mathcal{R} := \Big\{ r \in \mathbb{R}^{|N|} \Big| \; r \text{ is feasible for the real network} \Big\}$$

$$\mathcal{P}^{FB-EP} := \Big\{ p \in \mathbb{R}^{|Z|} \Big| \; \exists r \in \mathcal{R} : p_z = \sum_{n \in N(z)} r_n \quad \forall z \in Z \Big\}$$

4-node, 3-zone network: $p_A = r_1 + r_2, \; p_B = r_3, \; p_C = r_4$

Flow-Based Market Coupling with Exact Projection (FBMC-EP)

$$\mathcal{P}^{FB-EP} = \left\{ p \in \mathbb{R}^{|Z|} \,\middle|\, \exists(\bar{v}, f, \theta) \in [0,1]^{|G|} \times \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} : \right.$$

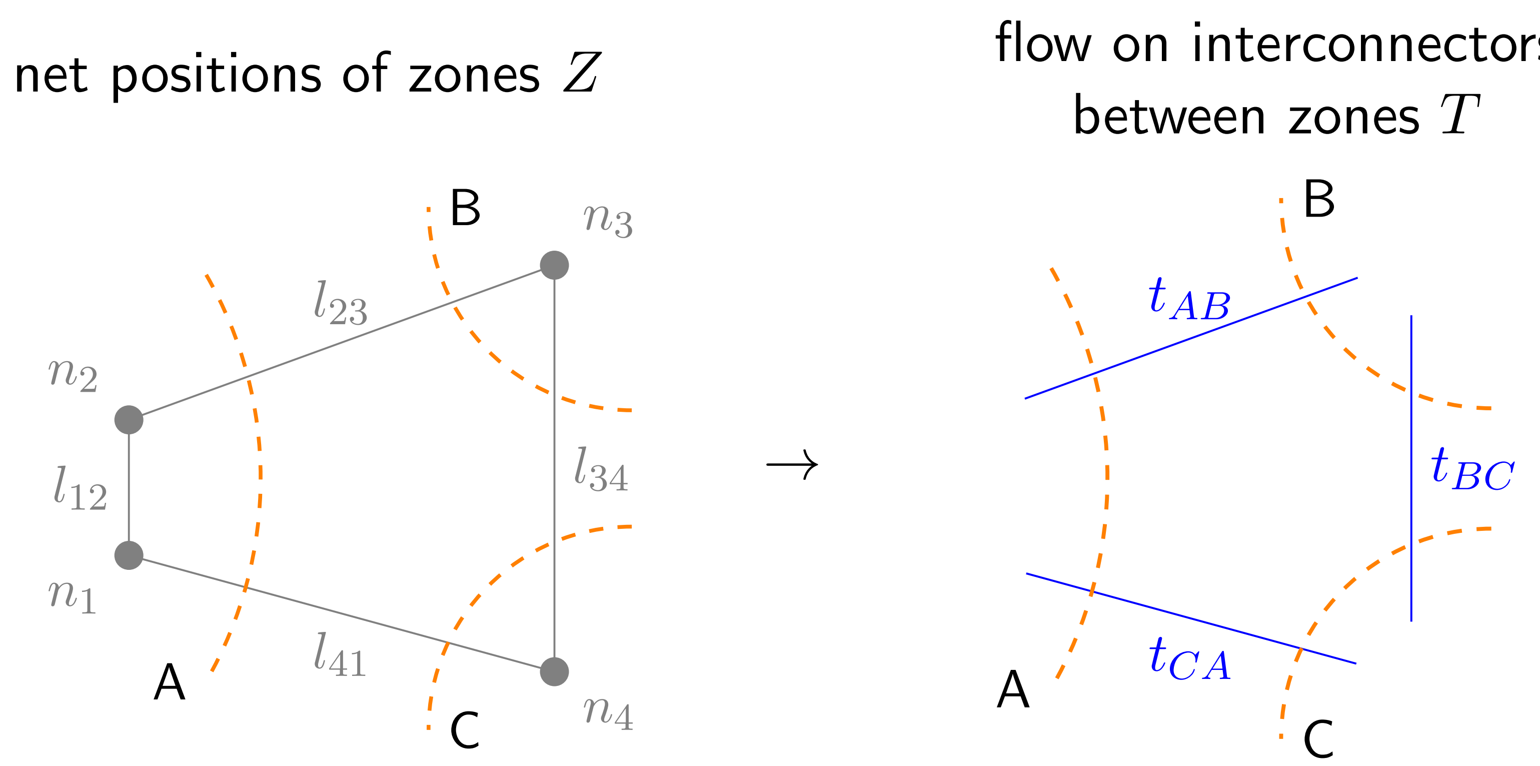
$$\sum_{g \in G(z)} Q_g \bar{v}_g - p_z = \sum_{n \in N(z)} Q_n \quad \forall z \in Z,$$

$$\sum_{g \in G(n)} Q_g \bar{v}_g - \sum_{l \in L(n,\cdot)} f_l + \sum_{l \in L(\cdot,n)} f_l = Q_n \quad \forall n \in N,$$

$$\left. -F_l \leq f_l \leq F_l, \; f_l = B_l \left(\theta_{m(l)} - \theta_{n(l)} \right) \quad \forall l \in L \right\}$$

- $\mathcal{P}^{FB-EP}$ **allows for all trades that are feasible** with respect to the real network and **bans only trades that can be proven to be infeasible** for the real network
- $\mathcal{P}^{FB-A}$ provides no guarantees: might ban feasible trades and, also, allow infeasible trades

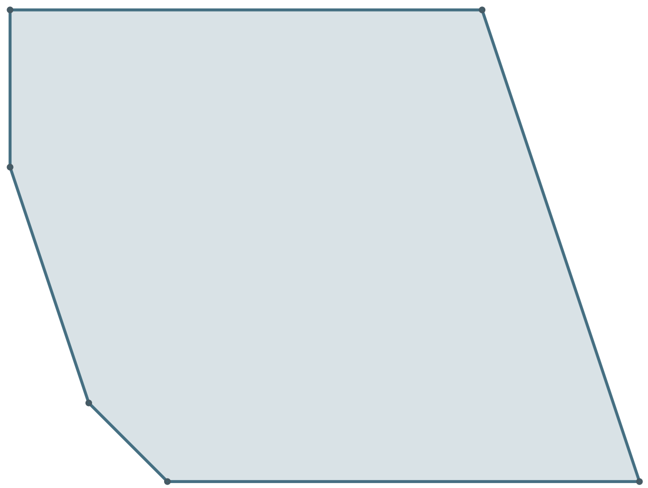
Available-Transfer-Capacity Market Coupling (ATCMC)



- ATCMC clears day-ahead market over the network $\mathcal{G}(Z, T)$
- Available-transfer-capacities (ATCs): $ATC_t^- \leq e_t \leq ATC_t^+$
- How to compute ATCs?

Available-Transfer-Capacity Market Coupling (ATCMC)

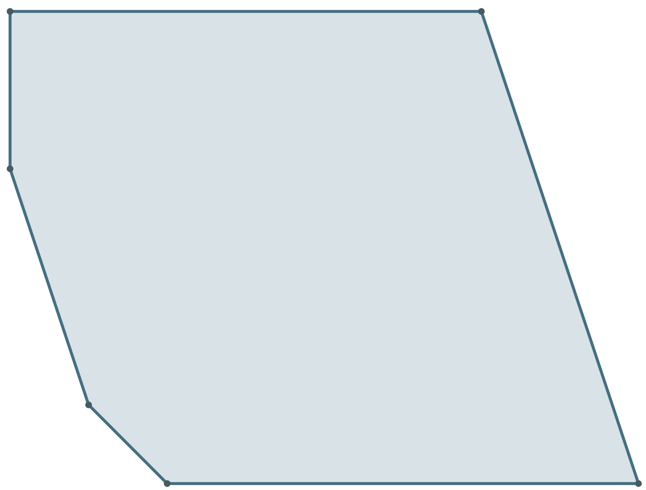
space of zonal net positions → space of exchanges



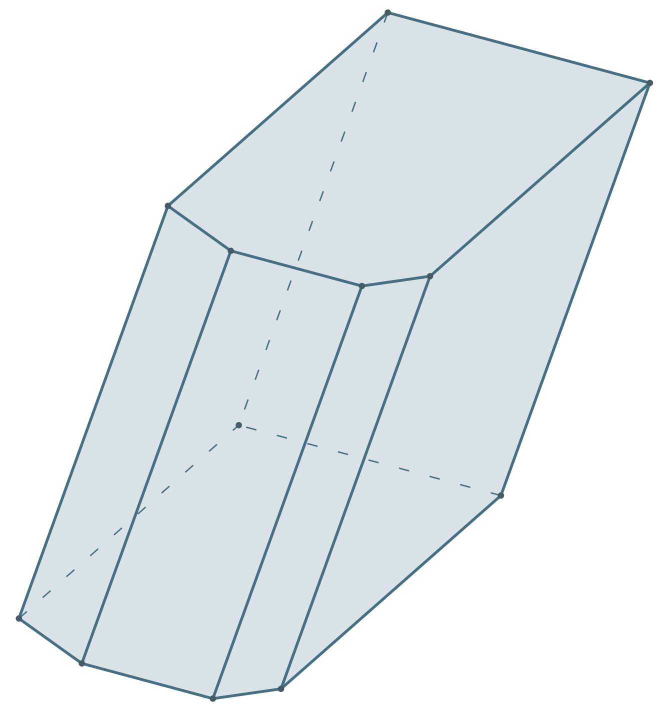
$\mathcal{P}^{FB-EP}$

Available-Transfer-Capacity Market Coupling (ATCMC)

space of zonal net positions $\rightarrow$ space of exchanges



$\mathcal{P}^{FB-EP}$



$$\mathcal{E} := \left\{ e \in \mathbb{R}^{|T|} \;\middle|\; -\sum_{l \in L(t)} F_l \leq e_t \leq \sum_{l \in L(t)} F_l \quad \forall t \in T, \right.$$
$$\left. \exists p \in \mathcal{P} : p_z = \sum_{t \in T(z,\cdot)} e_t - \sum_{t \in T(\cdot,z)} e_t \quad \forall z \in Z \right\}$$

Maximum-volume Available-Transfer-Capacities (ATCs)

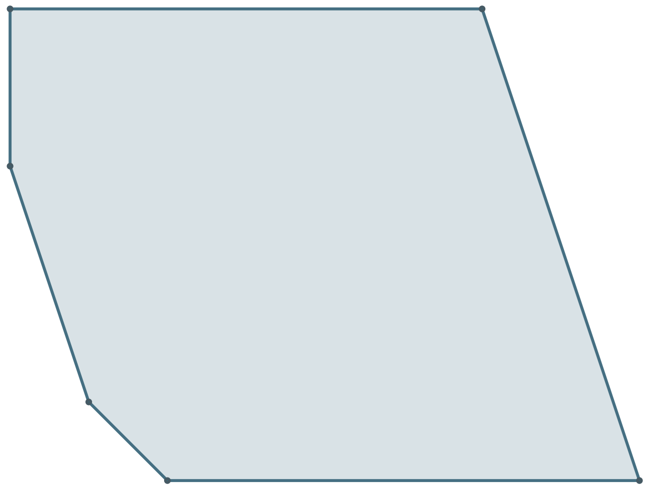
- ATC limits, $ATC_t^- \leq e_t \leq ATC_t^+ \ \forall t \in T$, are a box in the space of exchanges
- Compute ATCs as a maximum volume box inside $\mathcal{E}$,

$$\max_{ATC} \quad \prod_{t \in T} (ATC_t^- + ATC_t^+)$$
$$\text{s.t.} \quad [-ATC^-, ATC^+] \subseteq \mathcal{E}$$

- $\mathcal{E}^{ATC} := [-ATC^{-,*}, ATC^{+,*}]$ allows for the **largest subset of bilateral exchanges** that can be accommodated using a box and it **bans** all trades that would result in **infeasible zonal net positions**
- Implemented methodology for computing ATCs provides no guarantees

Maximum-volume Available-Transfer-Capacities (ATCs)

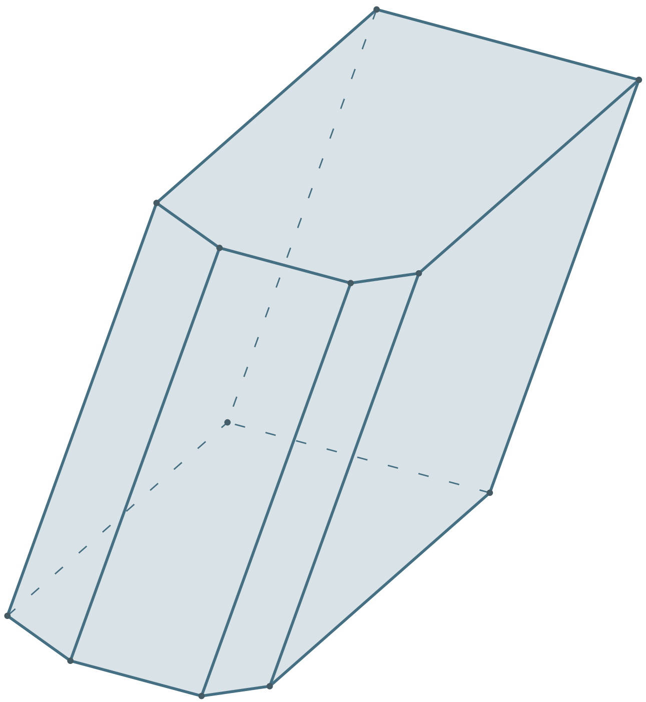
space of zonal net positions



$\mathcal{P}$

$\rightarrow$

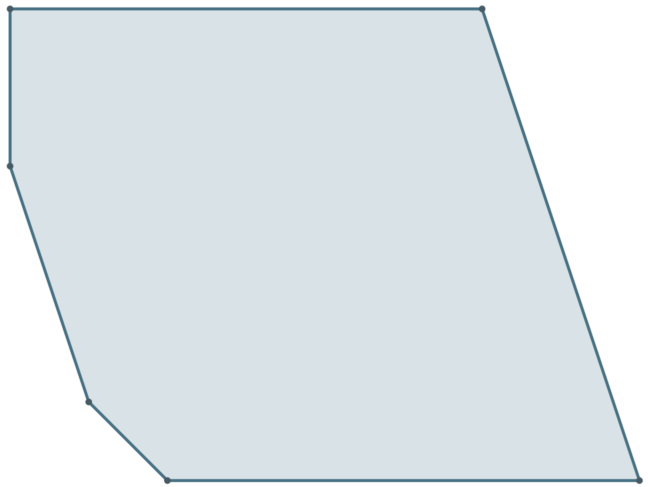
space of exchanges



$\mathcal{E}$

Maximum-volume Available-Transfer-Capacities (ATCs)

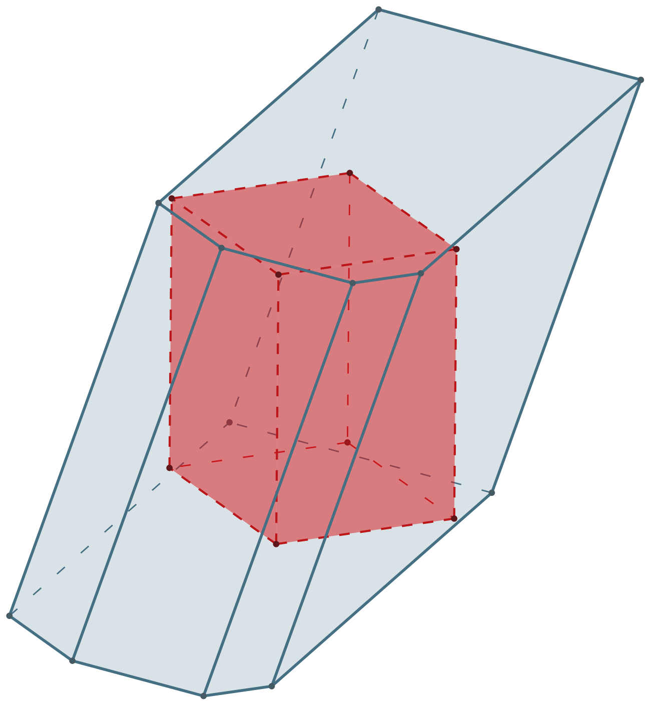
space of zonal net positions



$\mathcal{P}$

$\rightarrow$

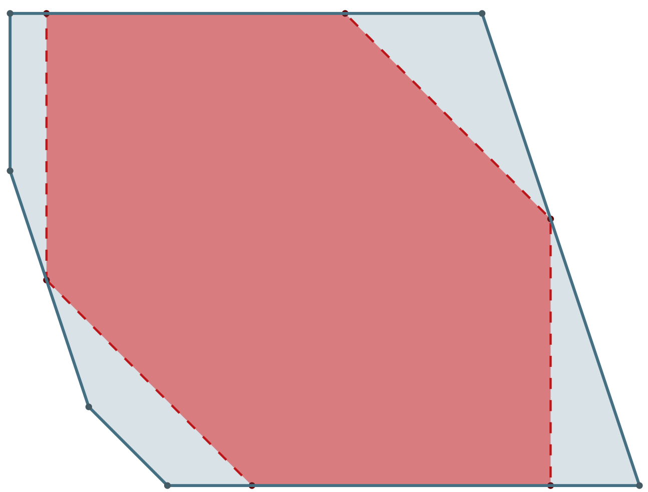
space of exchanges



$\mathcal{E}$, $\mathcal{E}^{ATC} = [-ATC^{-,*}, ATC^{+,*}]$

Maximum-volume Available-Transfer-Capacities (ATCs)

space of zonal net positions

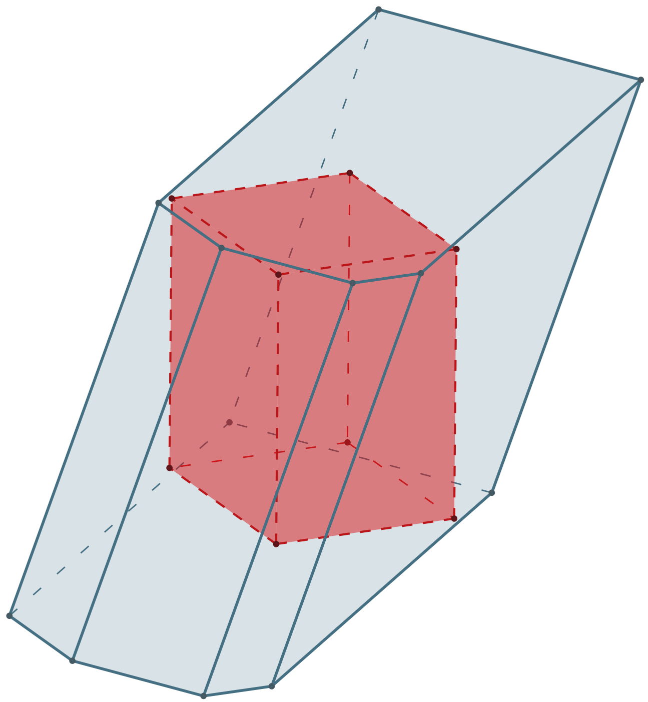


$\mathcal{P}$

$\rightarrow$

$\leftarrow$

space of exchanges



$$\mathcal{P}^{ATC} := \left\{ p \in \mathbb{R}^{|Z|} \middle| \exists e \in \mathcal{E}^{ATC} : p_z = \sum_{t \in T(z,\cdot)} e_t - \sum_{t \in T(\cdot,z)} e_t \ \forall z \in Z \right\}$$

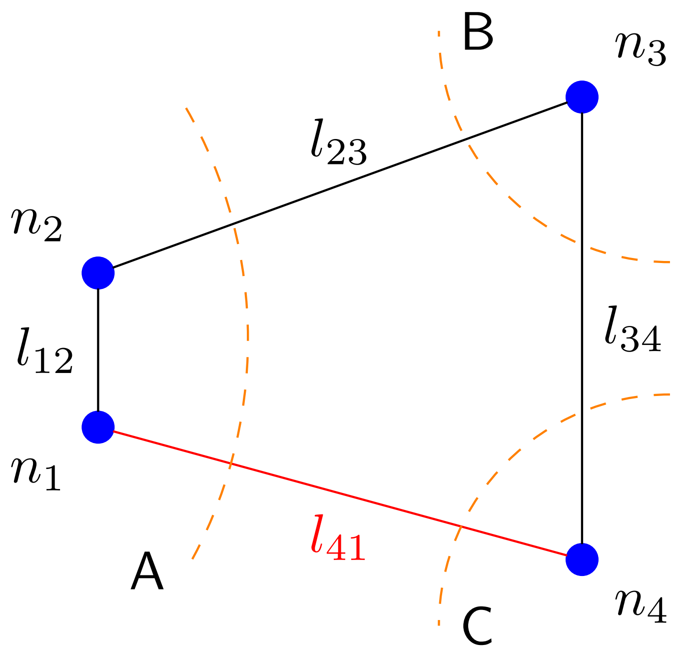
$$\mathcal{E},\ \mathcal{E}^{ATC} = [-ATC^{-,*}, ATC^{+,*}]$$

Policy analysis using 4-node, 3-zone network

Recap: 4 policies

- **LMP**: Minimizes cost over nodal model
- **FBMC-A**: Minimizes cost over zonal model with $\mathcal{P} = \mathcal{P}^{FB-A}$
- **FBMC-EP**: Minimizes cost over zonal model with $\mathcal{P} = \mathcal{P}^{FB-EP}$
- **ATCMC**: Minimizes cost over zonal model with $\mathcal{P} = \mathcal{P}^{ATC}$

Inter-zonal congestion case study



$$B_l = 1\text{pu}\ \forall l \in L$$

Generators

g	$n(g)$	Q_g [MW]	P_g [$/MWh]
1	n_1	500	8
2	n_2	200	45
3	n_3	300	18
4	n_4	500	200

Consumers

n	Q_n [MW]
n_2	300
n_4	300

$$F_{l_{41}} = 100\text{MW}, F_l = +\infty\ \forall l \in L \setminus \{l_{41}\}$$

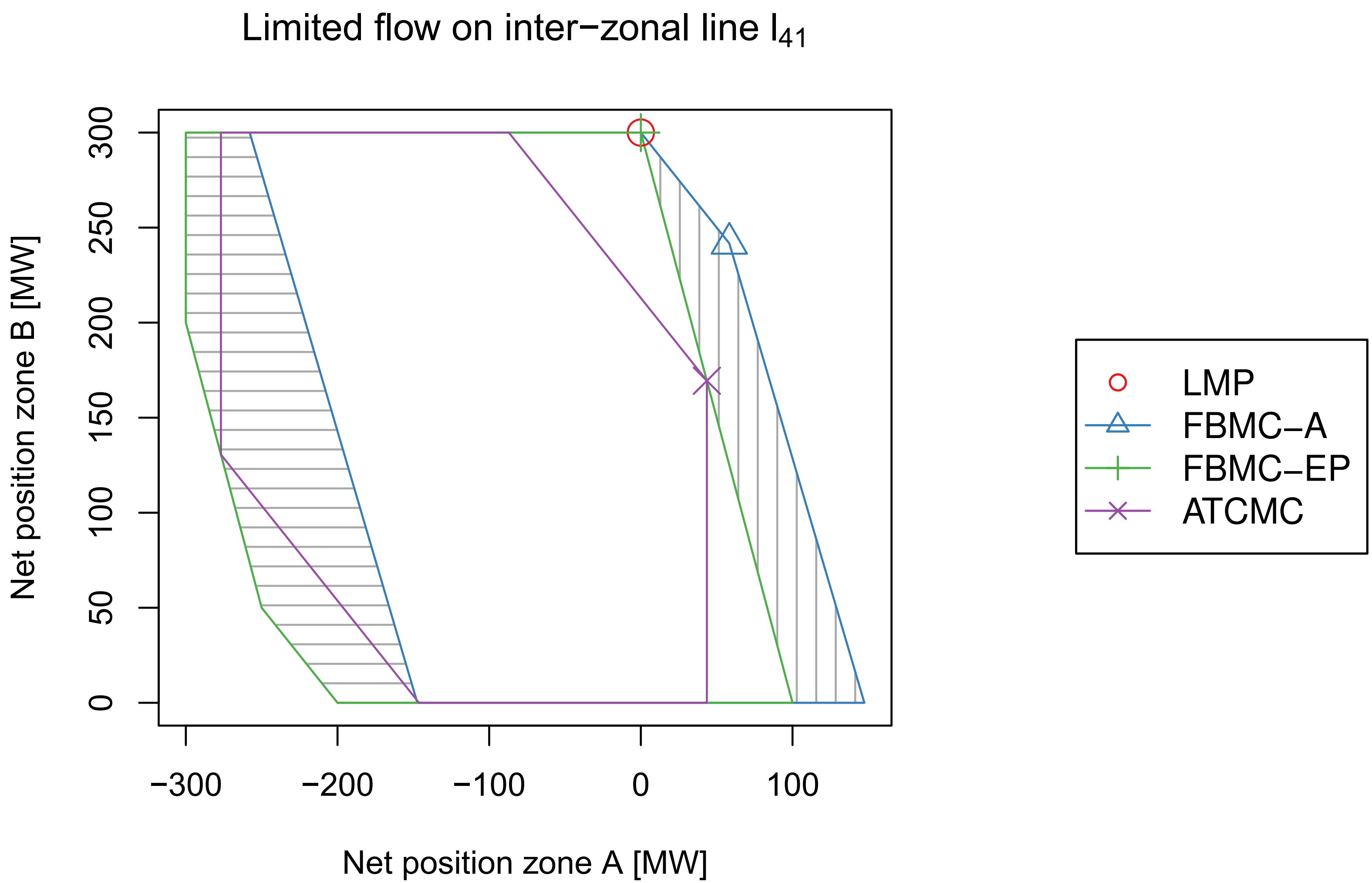
Inter-zonal congestion: results

Summary of clearing quantities and prices for a case of inter-zonal congestion (l_{41} limited to 100MW)

Policy	Total cost [$]	ρ [$/MWh]	Abs. error flow approx. [MW]	Overload l_{41} [MW]
LMP	15 200	(8, 45, 82, 119)	0	0
FBMC-A	7 217	(8, 18, 200)	475	79
FBMC-EP	7 800	(8, 18, 23)	300	50
ATCMC	23 208	(8, 18, 200)	–	50

- Cleared quantities on all zonal markets overload line l_{41}: **failure to account for inter-zonal congestion**
- Flow approximation error and overload larger in FBMC-A than in FBMC-EP
- Cost of ATCMC larger than cost of LMP, ATCMC clears at an infeasible point

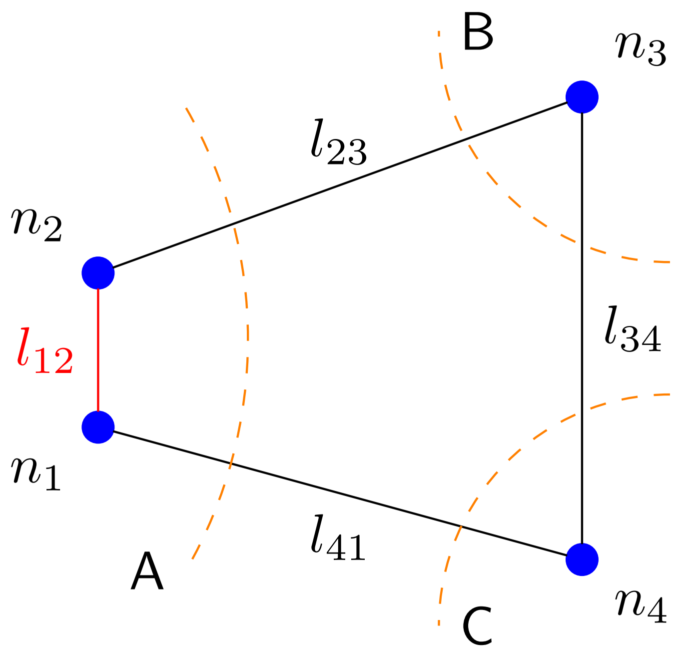
Inter-zonal congestion: space of zonal net positions



Vertically shaded area: $\mathcal{P}^{FB-A} - \mathcal{P}^{FB-EP}$

Horizontally shaded area: $\mathcal{P}^{FB-EP} - \mathcal{P}^{FB-A}$

Intra-zonal congestion case study



$$B_l = 1\text{pu } \forall l \in L$$

Generators

g	$n(g)$	Q_g [MW]	P_g [\$/MWh]
1	n_1	500	8
2	n_2	200	45
3	n_3	300	18
4	n_4	500	200

Consumers

n	Q_n [MW]
n_2	300
n_4	300

$$F_{l_{12}} = 100\text{MW}, F_l = +\infty \ \forall l \in L \setminus \{l_{12}\}$$

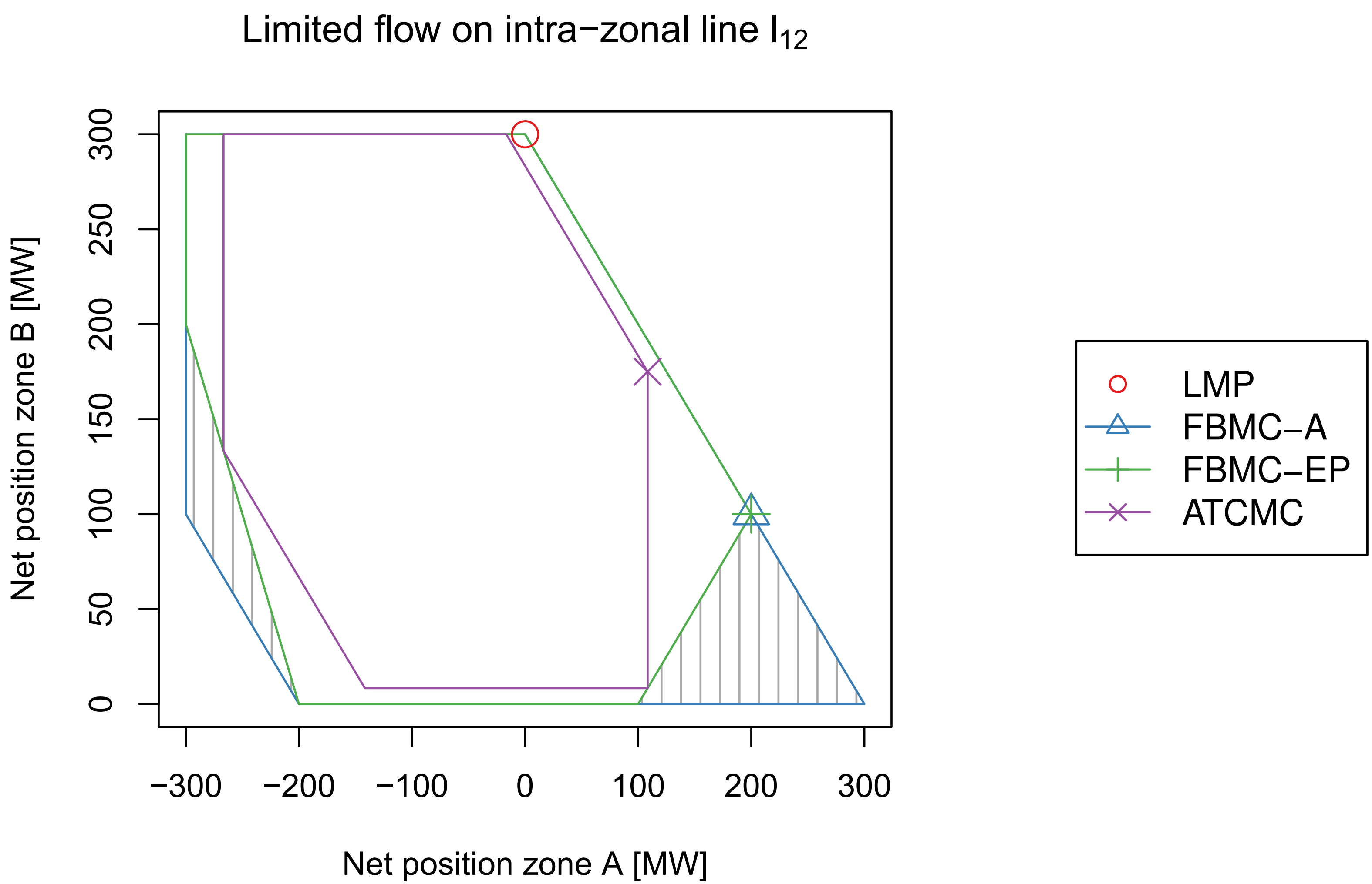
Intra-zonal congestion: results

Summary of clearing quantities and prices for a case of intra-zonal congestion (l_{12} limited to 100MW)

Policy	Total cost [$]	ρ [$/MWh]	Abs. error flow approx. [MW]	Overload l_{12} [MW]
LMP	10 267	(8, 45, 32.7, 20.3)	0	0
FBMC-A	5 800	(18, 18, 200)	536	150
FBMC-EP	5 800	(18, 18, 18)	300	150
ATCMC	9 750	(8, 18, 200)	–	108

- Cleared quantities on all zonal markets overload line l_{12}: **failure to account for intra-zonal congestion**
- Flow approximation error larger in FBMC-A than in FBMC-EP

Intra-zonal congestion: space of zonal net positions



Vertically shaded area: $\mathcal{P}^{FB-A} - \mathcal{P}^{FB-EP}$

Discussion

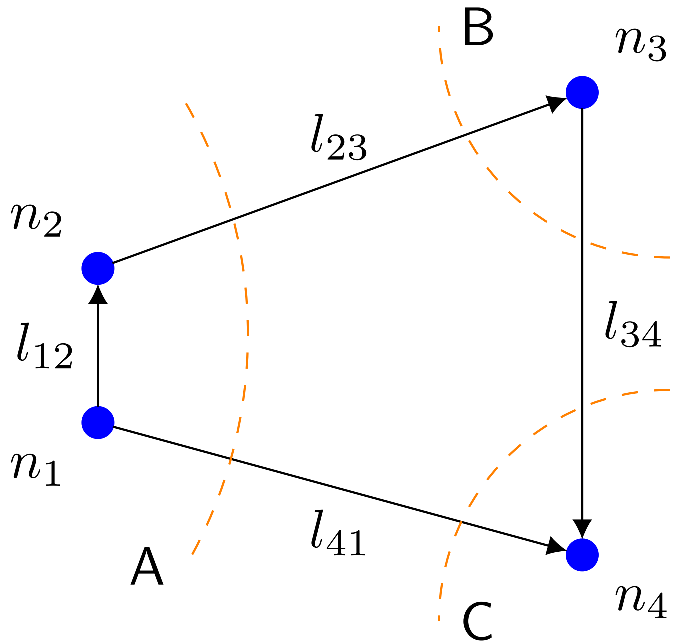
- FBMC-A, FBMC-EP, ATCMC led to clearing quantities that would overload the transmission system
- FBMC-A can be cleared with infeasible net positions and can prevent feasible trades from being accepted

  In direct conflict with EC 714/2009, Annex I, Art. 1.1 and 1.7
- **All zonal market designs suffer the same problem** for inter- and intra-zonal transmission capacity allocation: losing track of nodal injections leads to **inaccurate physical flow estimations**
- Given these results, in what follows, we only use FBMC-EP for modelling flow-based market coupling

Cutting-plane algorithms for robust day-ahead electricity markets

Nodal N-1 security criterion

- **Nodal net injections**, $r_n = \sum_{g \in G(n)} Q_g v_g - Q_n$, should be feasible even if a single transmission element is not available



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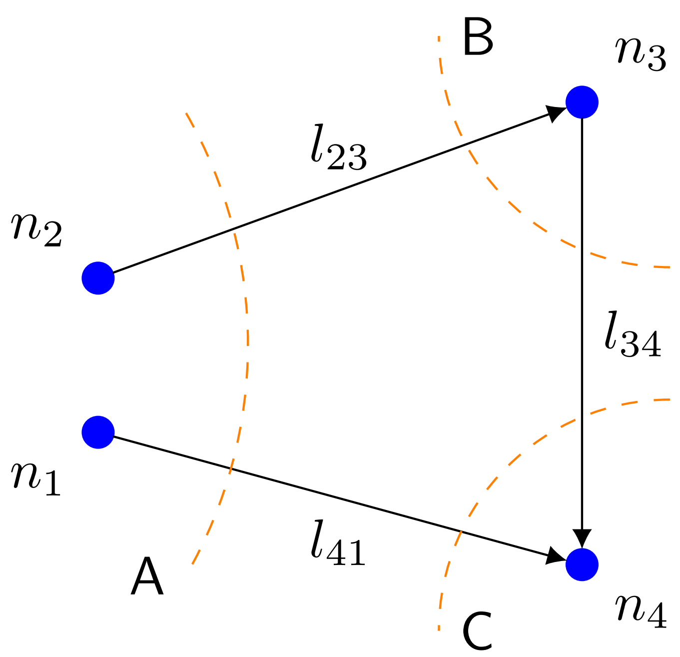

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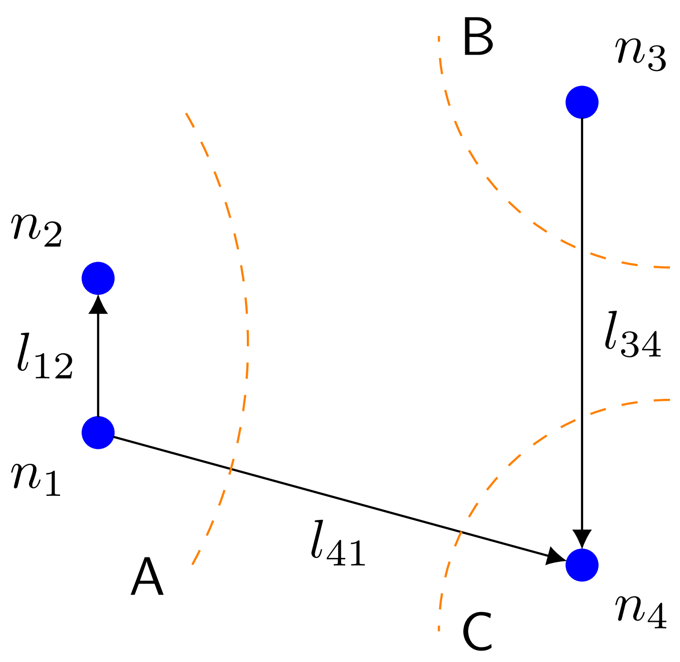


Generators

g	$n(g)$	Q_g [MW]	P_g [$/MWh]
1	n_1	500	8
2	n_2	200	45
3	n_3	300	18
4	n_4	500	200

Nodal day-ahead market clearing problem with N-1 security

$$\min_{v,r} \sum_{g \in G} P_g Q_g v_g$$

$$\text{s.t. } 0 \leq v_g \leq 1 \quad \forall g \in G$$

$$\sum_{g \in G(n)} Q_g v_g - Q_n = r_n \quad \forall n \in N \quad [\rho_n]$$

$$r \in \mathcal{R}_{N-1}$$

where $\mathcal{R}_{N-1} = \cap_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \mathcal{R}(u)$ and

$$\mathcal{R}(u) = \left\{ r \in \mathbb{R}^{|N|} \middle| \exists (f, \theta) \in \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} : \right.$$

$$r_n = \sum_{l \in L(n,\cdot)} f_l - \sum_{l \in L(\cdot,n)} f_l \ \forall n \in N,$$

$$\left. - F_l \leq f_l \leq F_l, \ f_l = B_l(1 - u_l)\left(\theta_{m(l)} - \theta_{n(l)}\right) \ \forall l \in L \right\}$$

Cutting-plane algorithm for FBMC under N-1 security

- $\mathcal{R}_{N-1}$ is a **convex polytope** $\rightarrow$ we can describe it as $\overline{V}r \leq \overline{W}$, for certain $\overline{V} \in \mathbb{R}^{M \times |N|}$ and $\overline{W} \in \mathbb{R}^{|N|}$
- $MCO(V, W)$ (Market Clearing Oracle): Clears day-ahead nodal market using $Vr \leq W$ as a description of $\mathcal{R}_{N-1}$
- $IO(r)$ (Injection Oracle): Checks if $r \in \mathcal{R}_{N-1}$, returning a separating hyperplane $v^\top r \leq w$ if $r \notin \mathcal{R}_{N-1}$

1: Initialize $V := 0_{1,|N|}, W := 0$, `inclusion` $:=$ `FALSE`
2: **while** `!inclusion` **do**
3: Call $MCO(V, W) \rightarrow r$
4: Call $IO(r) \rightarrow$ `inclusion`, (v, w)
5: $V := [V^\top \; v]^\top$, $W := [W^\top \; w]^\top$
6: **end while**
7: Terminate: inner model of $MCO(V, W)$ gives the optimal clearing.

Injection oracle (IO)

- Recall $\mathcal{R}_{N-1} = \bigcap_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \mathcal{R}(u)$
- Generating Benders' cuts independently for every $u \in \{0,1\}^{|L|}$, $\|u\|_1 \leq 1$ can be very expensive
- A better approach: use a **distance function** d between the query point r and the set $\mathcal{R}_{N-1}$

$$d(r, \mathcal{R}_{N-1}) := \max_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \min_{\bar{r} \in \mathcal{R}(u)} \|r - \bar{r}\|_1$$

  which can be evaluated by solving a single-level MILP
- Cutting-plane algorithm converges finitely
- Algorithm inspired by original work by Street *et al.* (2014) on security constrained unit commitment

Zonal day-ahead market clearing problem with N-1 security

- **Zonal net positions** should be feasible even if a single transmission element is not available

$$
\begin{aligned}
\min_{v,p} \quad & \sum_{g \in G} P_g Q_g v_g \\
\text{s.t.} \quad & 0 \leq v_g \leq 1 \quad \forall g \in G \\
& \sum_{g \in G(z)} Q_g v_g - \sum_{n \in N(z)} Q_n = p_z \quad \forall z \in Z \quad [\rho_z] \\
& p \in \mathcal{P}
\end{aligned}
$$

- **FBMC**: $\mathcal{P} \equiv \mathcal{P}_{N-1}^{FB-EP} := \{p \in \mathbb{R}^{|Z|} \mid$ there exists a feasible $(\bar{v}, f, \theta)$ for each single-element unavailability scenario$\}$

- **ATCMC**: $\mathcal{P} \equiv$ projection of ATC box, computed as maximum-volume box inside $\mathcal{E}_{N-1}$ (defined in an analogous fashion to $\mathcal{E}$)

Set of feasible net positions under N-1 security

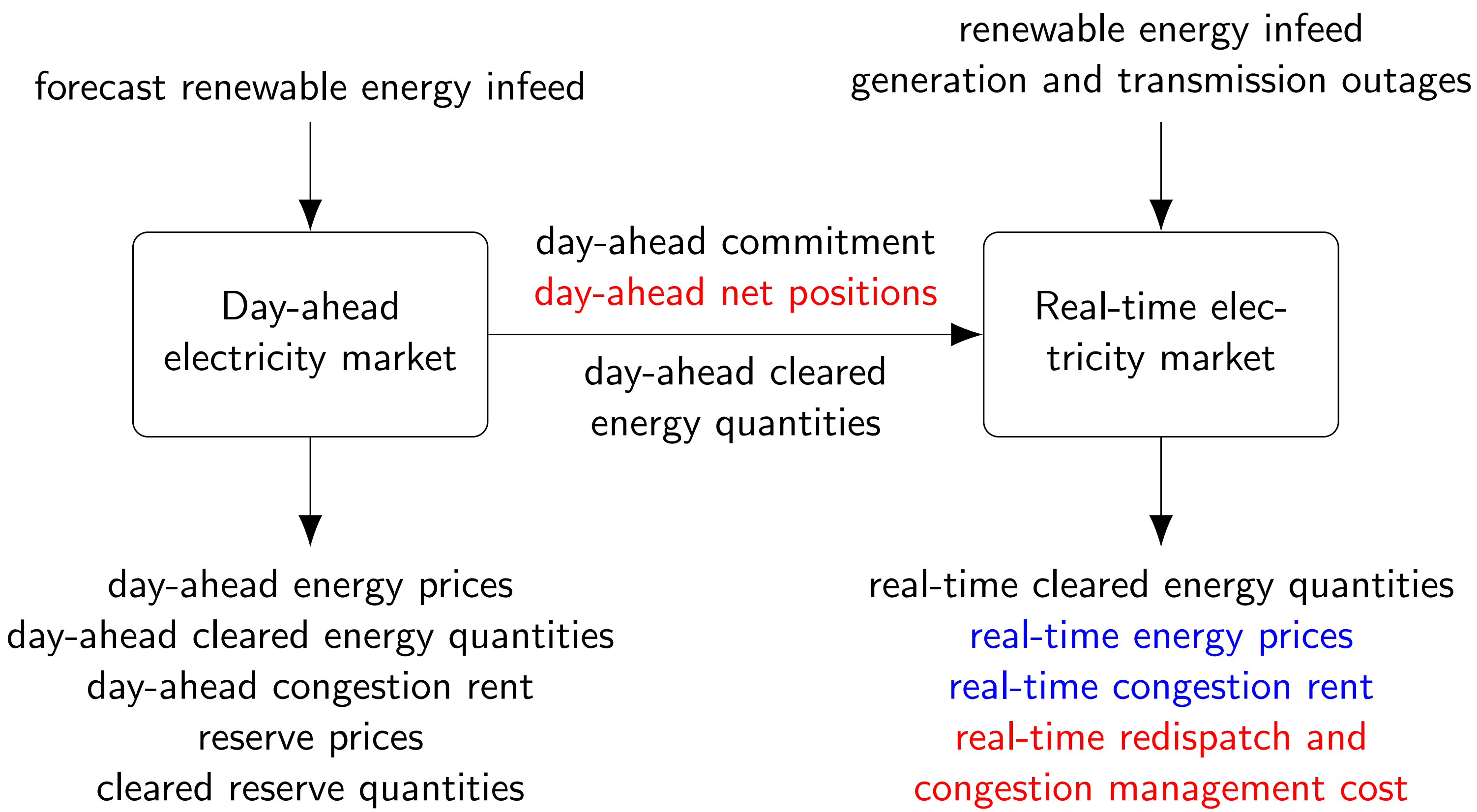
- Define $\mathcal{P}_{N-1}^{FB-EP} := \cap_{\substack{u \in \{0,1\}^{|L|} \\ \|u\|_1 \leq 1}} \mathcal{P}^{FB-EP}(u)$, where

$$\mathcal{P}^{FB-EP}(u) = \left\{ p \in \mathbb{R}^{|Z|} \middle| \exists(\bar{v}, f, \theta) \in [0,1]^{|G|} \times \mathbb{R}^{|N|} \times \mathbb{R}^{|L|} : \sum_{g \in G(z)} Q_g \bar{v}_g - \sum_{n \in N(z)} Q_n = p_z \ \forall z \in Z, \sum_{i \in G(n)} Q_g \bar{v}_g - Q_n = \sum_{l \in L(n,\cdot)} f_l - \sum_{l \in L(\cdot,n)} f_l \ \forall n \in N, -F_l \leq f_l \leq F_l,\ f_l = B_l(1-u_l)\left(\theta_{m(l)} - \theta_{n(l)}\right) \ \forall l \in L \right\}$$

- $\mathcal{P}_{N-1}^{FB-EP}$ is a **convex polytope** $\rightarrow$ we can clear FBMC using an analogous cutting-plane approach to that used for LMP
- Computing max-volume ATCs: similar approach, but separating hyperplanes guarantee box inclusion instead of point inclusion

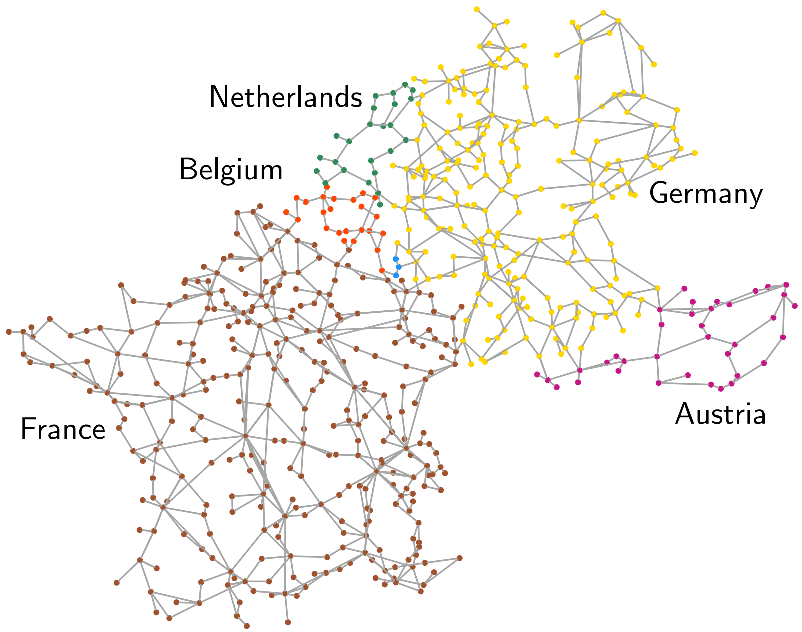
Simulation results for the Central Western European network

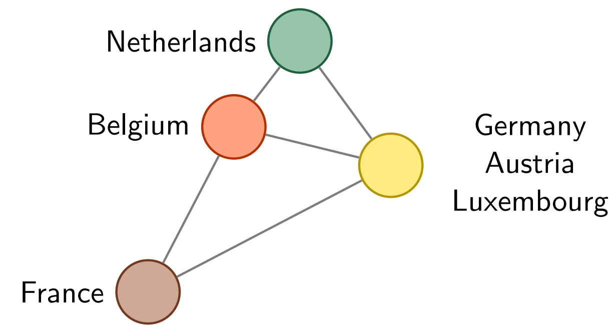
Two-settlement system



- Commitment refers to $\{0, 1\}$ (on-off) decisions $\rightarrow$ European rules for integer pricing, Madani and Van Vyve (2015)

Central Western European network





- 632 buses, 945 branches (3 491 individual circuits), 346 slow thermal generators (154GW), 301 fast thermal generators (89GW) and 1 312 renewable generators (149GW)
- 768 typical snapshots $\times$ 1 150 random uncertainty realizations $\rightarrow$ $\sim$100 years of operation

Implementation and deployment on HPC infrastructure

- Implementation in Julia/JuMP, using Gurobi, Ipopt compiled with HSL and Xpress as mathematical programming solvers
- High-performance computing (HPC) deployment using Julia's built-in parallel computing capabilities
- Total simulation time: ∼7 579 CPU-hours

Cab

Lawrence Livermore National Laboratory

Lemaitre3

Consortium des Équipements de Calcul Intensif

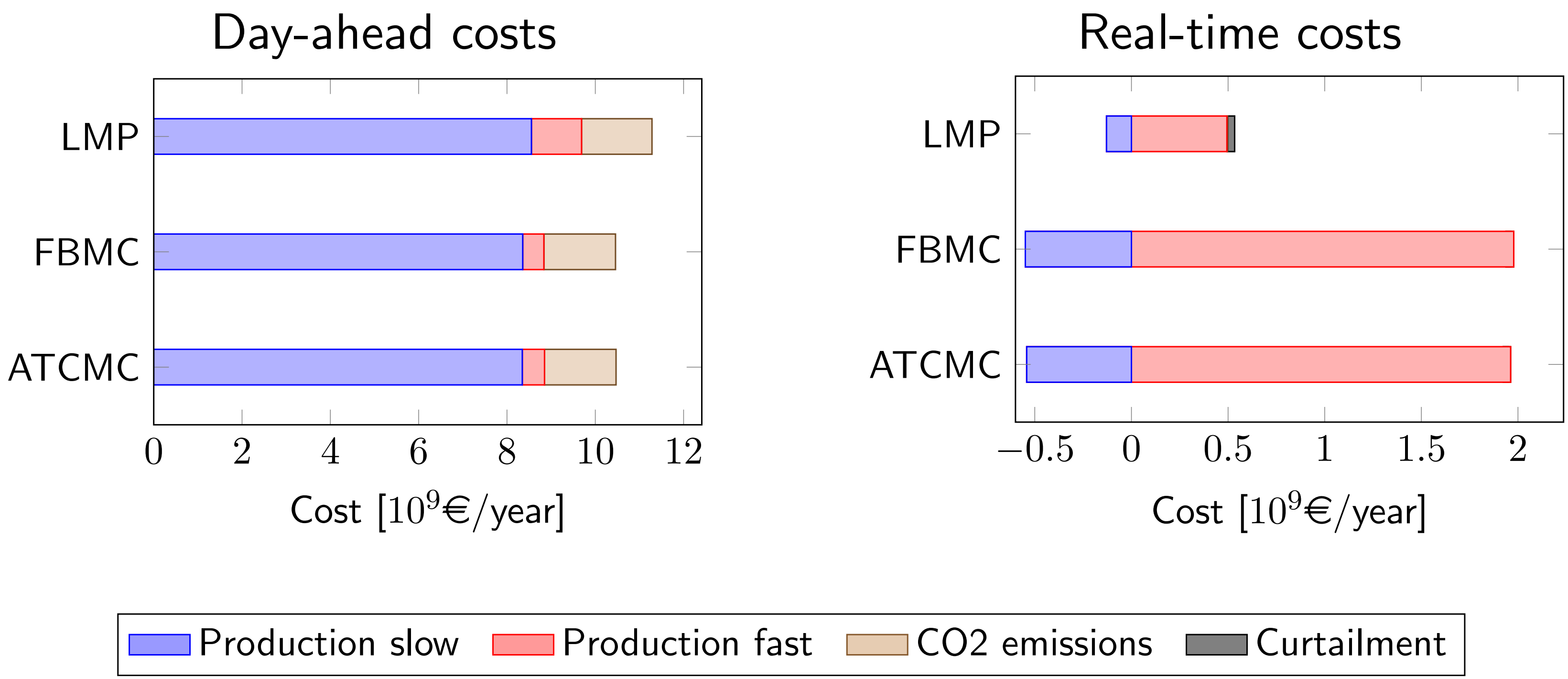
CWE results: total cost performance

Total costs and efficiency of different policies

Policy	Day-ahead [M€/year]	Real-time [M€/year]	Total [M€/year]	Efficiency losses
PF	–	11 476	11 476	-2.90%
LMP	11 284	534	11 818	–
FBMC	10 458	1 963	12 420	5.09%
ATCMC	10 470	1 949	12 419	5.08%

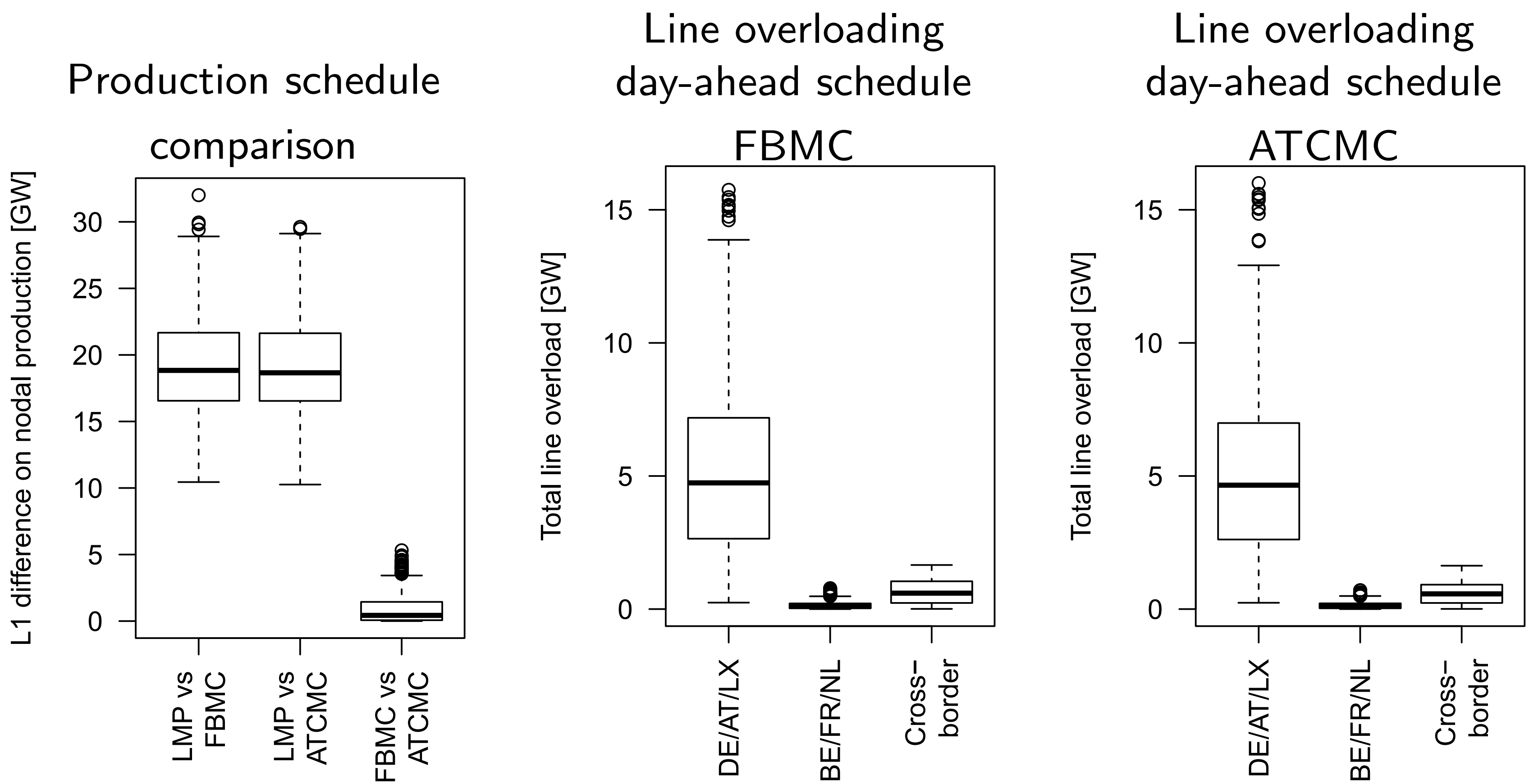
- PF: Perfect Foresight benchmark
- LMP schedules out-of-merit production to meet N-1 security, leading to inefficiencies with respect to PF
- Efficiency losses of zonal markets with respect to LMP amount to about 5.1% of total costs, ∼**600M€/year**
- FBMC and ATCMC **do not present a significant difference**

CWE results: costs composition



- Day-ahead costs larger for LMP, real-time costs much larger for FBMC and ATCMC than LMP
- Zonal policies: re-dispatching slow (cheap) generators down and re-dispatching fast (expensive) generators up in real time

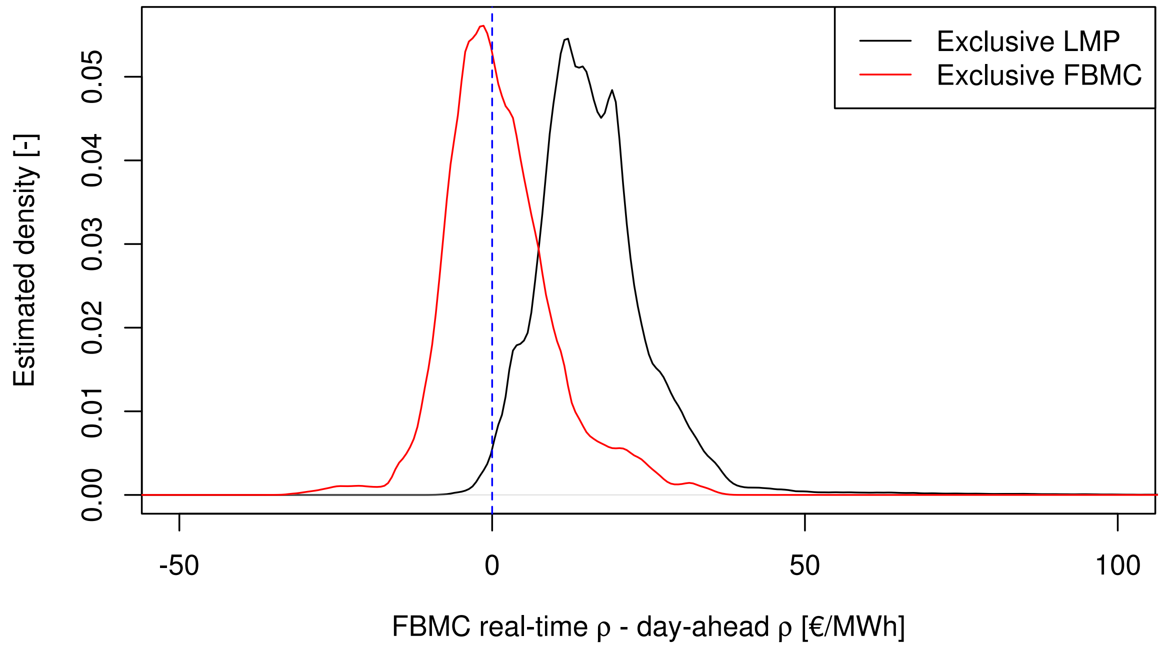
CWE results: production schedules and line overloading



- **Large differences** between day-ahead schedules of **nodal and zonal** policies, **small differences** between **FBMC and ATCMC**
- Zonal policies lead to decisions **overloading inter- and intra-zonal transmission lines**

CWE results: difference on commitment decisions

Price change in FBMC at nodes where $\text{UC}^{\text{FBMC}} \neq \text{UC}^{\text{LMP}}$



- "Exclusive LMP": weighted average change $\rho_n^{RT} - \rho_{z(n)}^{DA}$ over all nodes where LMP committed slow units and FBMC did not commit units. "Exclusive FBMC" series computed analogously.
- LMP commits capacity where it is needed. FBMC suffers from suboptimal commitment.

Conclusions

Conclusions

- **New framework for modelling zonal electricity markets**:
  - Projecting network constraints onto space of exports/imports
  - Free from discretionary parameters (base case, flow approximation, etc.)
- ATCMC and FBMC fail at allocating inter- and intra-zonal transmission capacity
- CWE: **ATCMC and FBMC do not present significant performance differences**
- CWE: **Nodal design outperforms ATCMC and FBMC** by ∼**600M€/year**

Thank you

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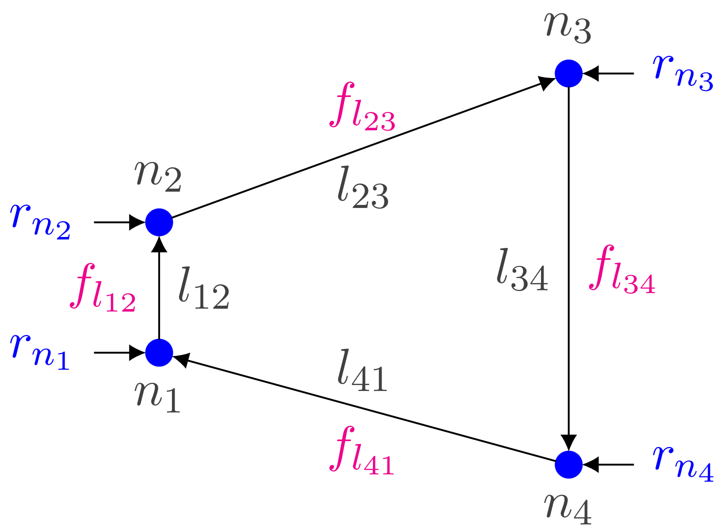
Appendix

Future extensions

- Application of the developed framework to answer policy questions in European electricty markets via simulation
- Extensions of the proposed framework:
  - Pricing AC power flow constraints implicitly on active power
  - TSO-DSO coordination

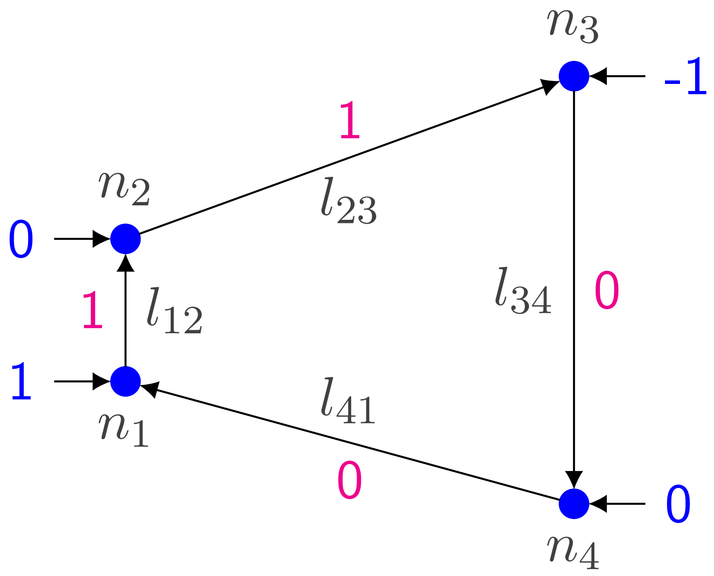
Electricity markets 101

Transportation networks



- Paths are decision variables
- Must respect nodal balance constraints ($\forall n \in N$)
$$r_n = \sum_{l \in L(n,\cdot)} f_l - \sum_{l \in L(\cdot,n)} f_l$$
- Must respect capacities of branches ($\forall l \in L$):
$$-F_l \leq f_l \leq F_l$$
- Note: $\sum_{n \in N} r_n = 0$

Transportation networks



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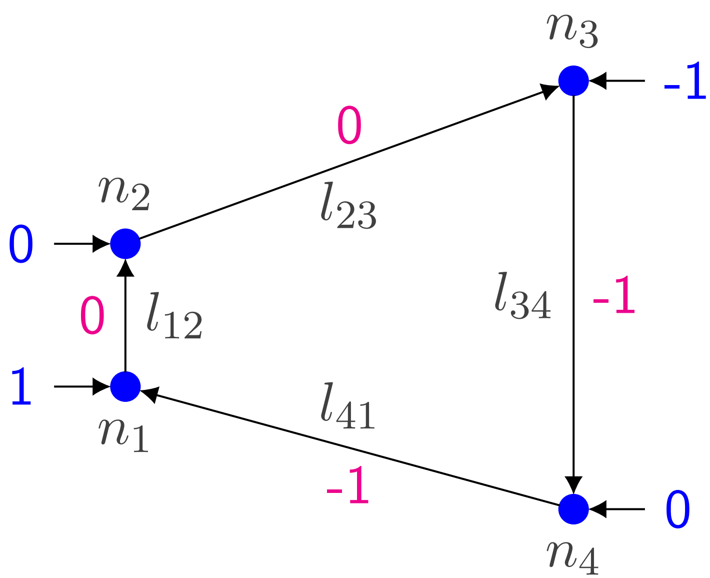

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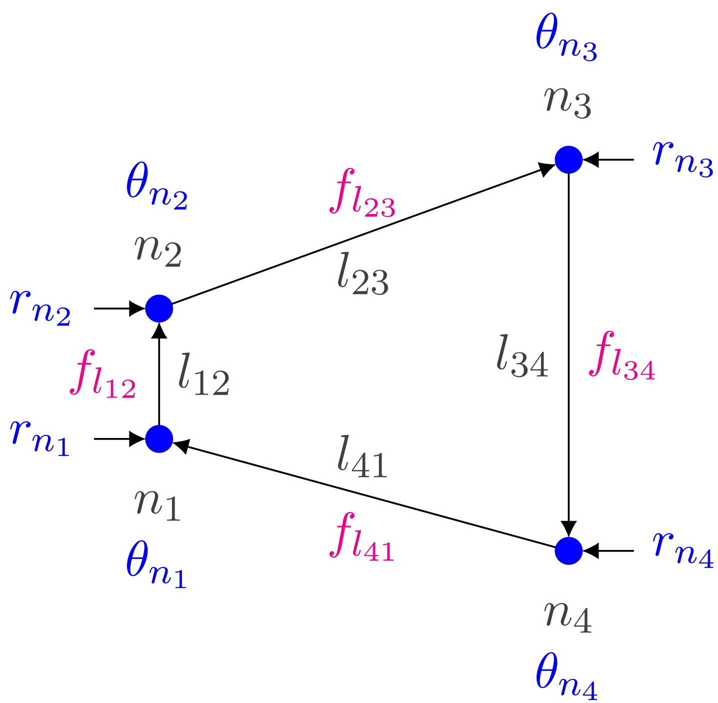
Electrical networks



- **Flow paths are implied by injections**
- Must respect nodal balance constraints ($\forall n \in N$)

$$r_n = \sum_{l \in L(n,\cdot)} f_l - \sum_{l \in L(\cdot,n)} f_l$$

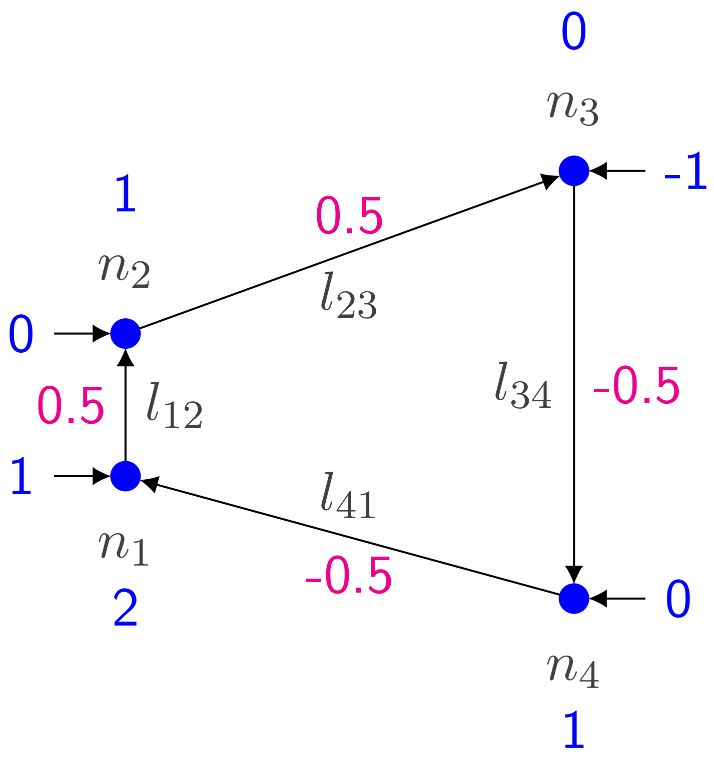
- Must respect capacities of branches ($\forall l \in L$):

$$-F_l \leq f_l \leq F_l$$

- Must respect power-angle constraints ($\forall l \in L$, branch l from $m(l)$ to $n(l)$):

$$f_l = B_l \left(\theta_{m(l)} - \theta_{n(l)}\right)$$

Electrical networks



$$B_{l_{12}} = B_{l_{23}} = B_{l_{34}} = B_{l_{41}} = 0.5$$

- **Flow paths are implied by injections**
- Must respect nodal balance constraints ($\forall n \in N$)

$$r_n = \sum_{l \in L(n,\cdot)} f_l - \sum_{l \in L(\cdot,n)} f_l$$

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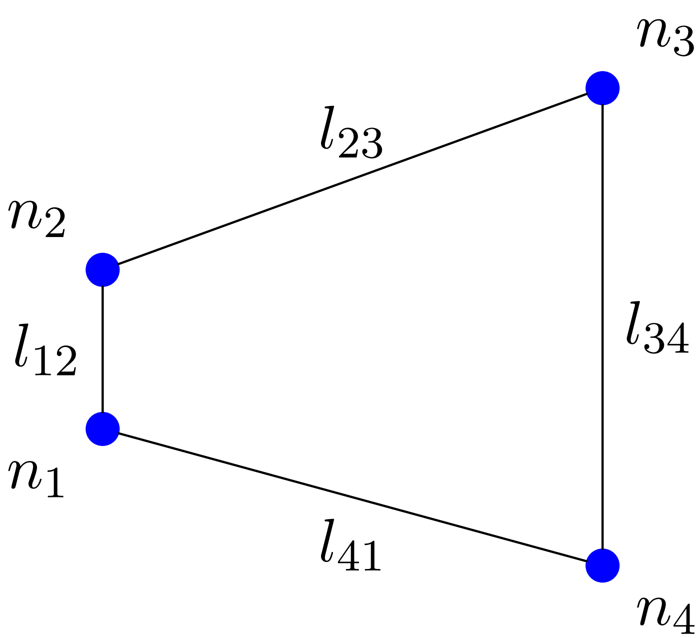
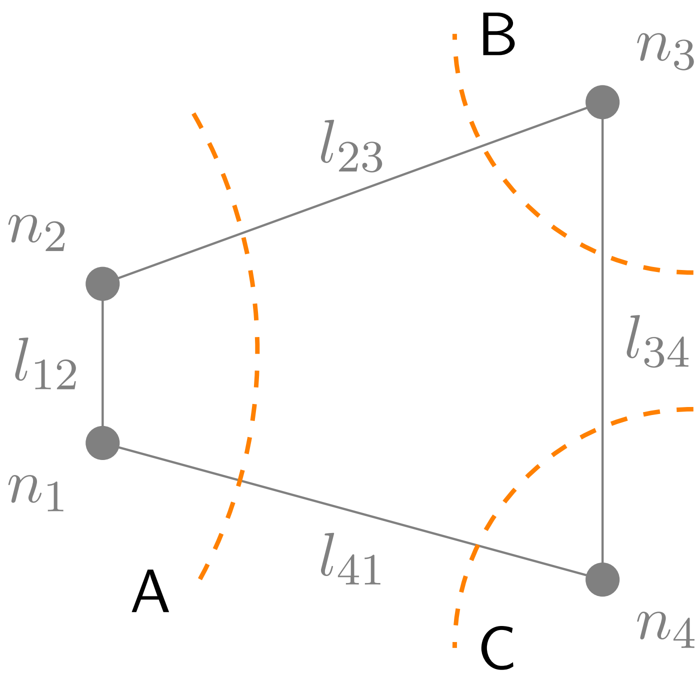
Nodal markets: trading power over an electrical network



- Participants can **trade freely** when located at the **same node**
- Participants in **different nodes** can trade up to **congestion** of lines
- Congestion: power flow equations, thermal limits of transmission lines, among others (**physics**)

Zonal markets: trading power over an aggregated network



- Participants **trade freely** within each **bidding zone**
- Participants in **different zones** can trade up to certain **export/import limits**
- Limits: **aggregation** of power flow equations and others