# Planning Energy Investment under Uncertainty

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joint with Felipe Atenas, University of Chile

**Cambridge, MESW02, March 19 2019**

**Electricity systems of the future:**
**incentives, regulation and analysis for efficient investment**

# Towards sustainable energy systems

- Sustainability concerns have increased the participation of renewable sources like wind and sun in electrical systems
- How to blend the different technologies to support the transition to a low carbon energy system?
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**Generation Expansion Planning**

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Long term problem (10 years or more)

Minimize investment and operational costs s.t.

- satisfaction of demand (unknown)
- financial constraints: construction times, delays (unknown)
- environmental constraints: limits on carbon emissions (unknown)
- operational data: fuel costs (unknown)
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**Given:**

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# Why environmental constraints?

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**BUT:**

- They are already in the mix
- They can be built by private companies (“cheap and fast”)
- Peakers respond fast to sudden operational changes
- Gas fueled plants come with technology to reduce emissions

# Some notation and elements of the model

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- Investment decisions $x$, generation decisions $y$
- Multidimensional uncertainty $\xi$
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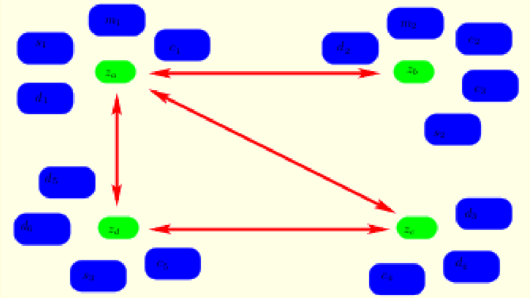
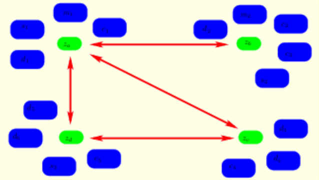
- Interconnected zones
- Zone balancing of supply and demand (inelastic)



# Important model features



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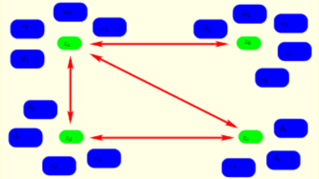

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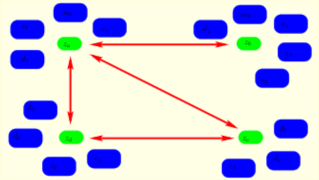

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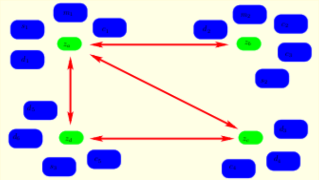

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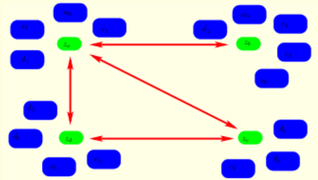

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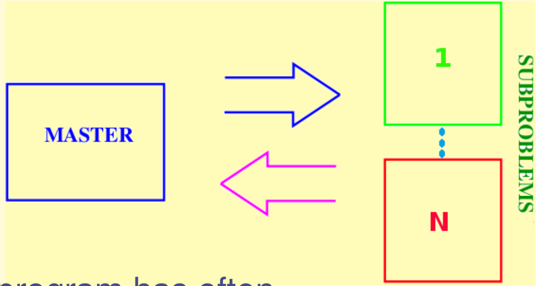
**Decomposition techniques**

# Decomposition principle

if a problem is difficult to solve directly,

solve instead a sequence of easier subproblems.

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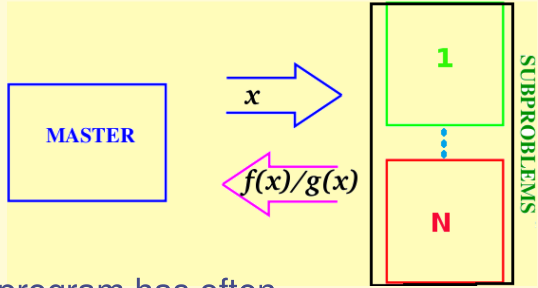


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Separate **subproblems** allow for fast oracle calculations

(the sum of *N* terms)

# Features preventing separability in the GEP

- **Spatial coupling:**
- **Temporal coupling:**
- **Stagewise coupling:**
- **Scenario coupling:**

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  investment and generation decisions
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  value-at-risk

# Features preventing separability in the GEP

- **Spatial coupling:** **demand constraint**

  all generation assets at each time step: $\sum_p y_p^t(\xi) \geq d^t(\xi)$, for each $t$

- **Temporal coupling:** **ramps, water balance**

  one/few generation assets: $\left(y_p^1(\xi), \ldots, y_p^T(\xi)\right) \in \mathcal{D}_p(\xi)$, for each $p$

- **Stagewise coupling:** **capacity constraint**

  investment and generation decisions: $y_p^t(\xi) \leq cap_p + x$, for each $t, p$

- **Scenario coupling:** **induced by risk aversion**

  value-at-risk: **VaR(gen.cost) couples ALL** $y(\xi)$

# Mathematical Model

(2-stage for simplicity)

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\text{with } CVaR_\alpha(X) &= \min\left\{u + \tfrac{1}{1-\alpha}\mathbb{E}[\max(0, X-u)] : u \in \mathbb{R}\right\} \\
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# Uncoupling stages 

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- Reformulates an expected-value objective function,

$$\mathbb{E}[c^\top x + q(\xi)^\top y(\xi)] = \sum_{\xi} p(\xi)\Big(c^\top x + q(\xi)^\top y(\xi)\Big)$$

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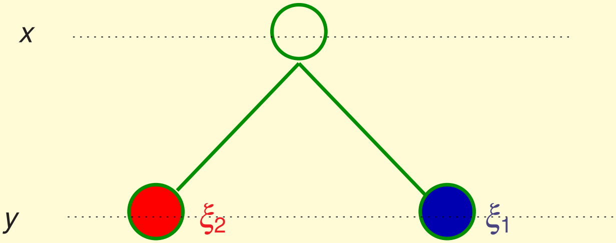

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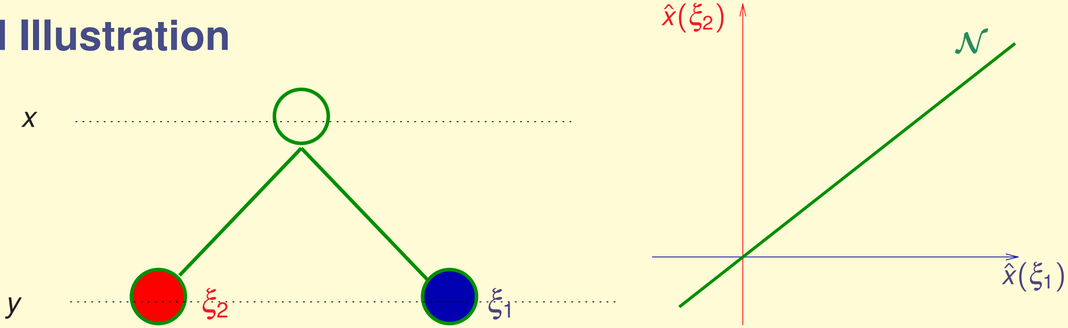

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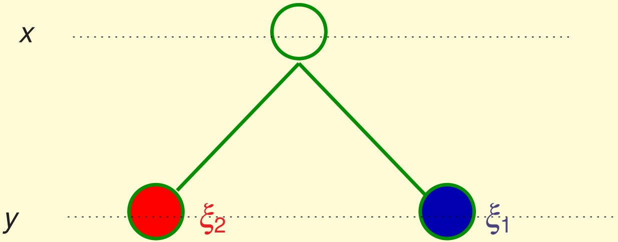


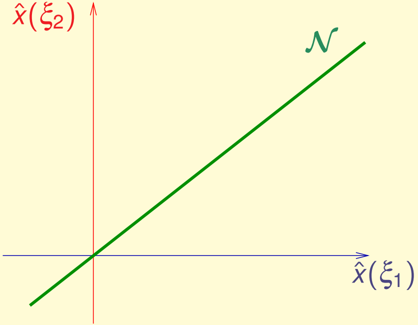


- Nonanticipativity constraint is a projection $x(\xi) = Proj_{\mathcal{N}}(\hat{x}(\xi))$
- On the linear subspace $\mathcal{N} := \{(x_1, x_2) : x_1 = x_2\}$ projection is **explicit**

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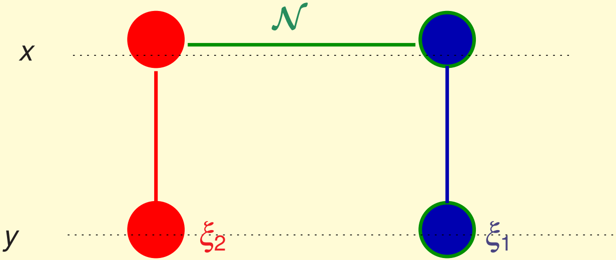


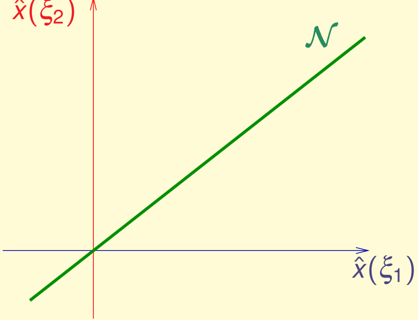


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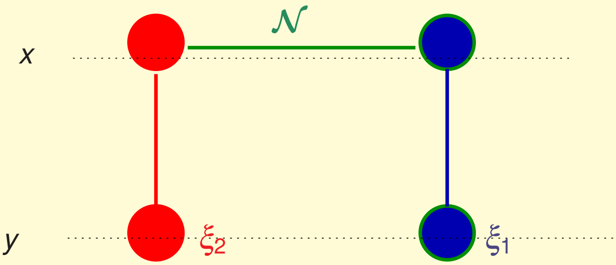


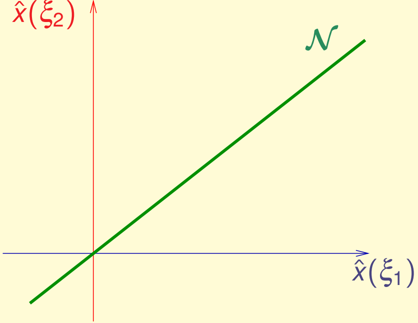


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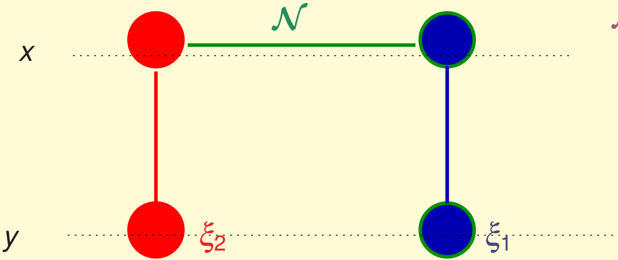


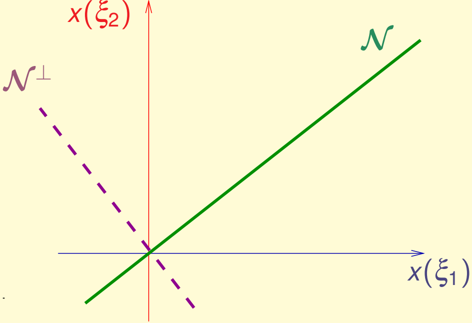


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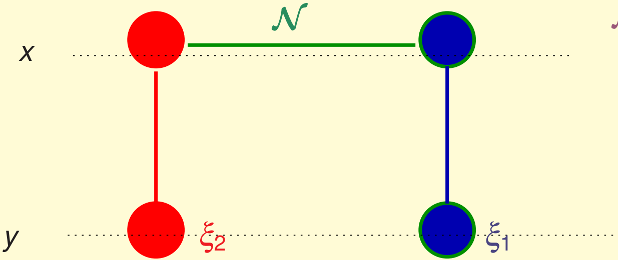


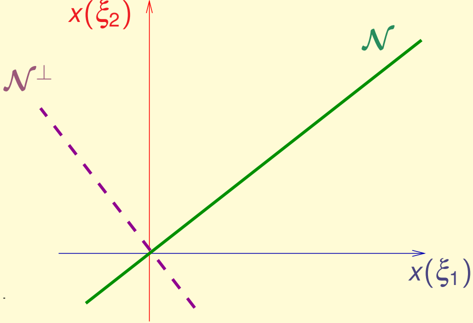


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# PH algorithm with 2 stages

Finds saddle point $(x^*, w^*)$ of Lagrangian

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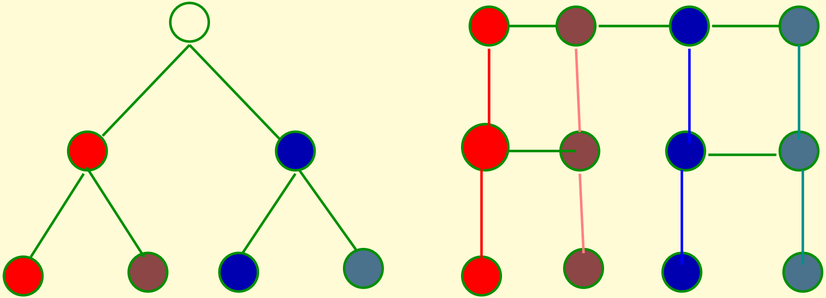
# PH for more than 2 stages

Same reasoning



- $x_t = Proj_{\mathcal{N}}(x_t) \iff x_t = \mathbb{E}_{\xi|\xi_{[t]}}[\hat{x}_t]$
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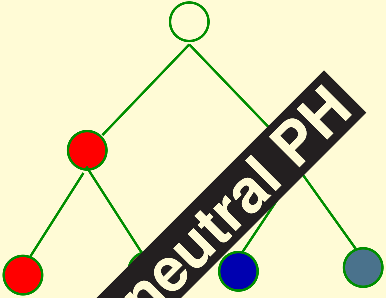

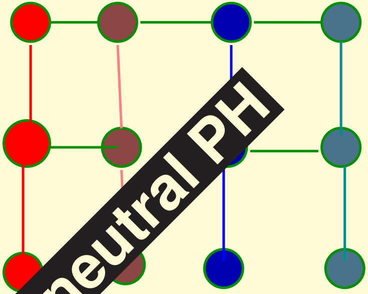

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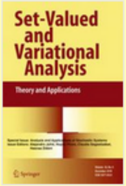
# Uncoupling stages  : +30 years later for CVaR!




Set-Valued and Variational Analysis

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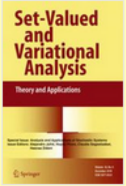


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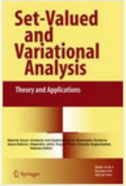
R. Tyrrell Rockafellar

- Creates copies $u(\xi)$ for the VaR variable, $u$, sets

$$u(\xi) = u \quad \text{a.e.}\xi,$$

- and relaxes the nonanticipativity constraint à la Augmented Lagrangian

# Uncoupling stages  : +30 years later for CVaR!

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## Solving Stochastic Programming Problems with Risk Measures by Progressive Hedging

Authors Authors and affiliations

R. Tyrrell Rockafellar

Convergence properties: the primal-dual error sequence

$$\left\{ \left\| \begin{pmatrix} x^k(\cdot) \\ u^k(\cdot) \end{pmatrix} - \begin{pmatrix} x^*(\cdot) \\ u^*(\cdot) \end{pmatrix} \right\|^2 + \frac{1}{r^2} \left\| \begin{pmatrix} w^k(\cdot) \\ w_u^k(\cdot) \end{pmatrix} - \begin{pmatrix} w^*(\cdot) \\ w_u^*(\cdot) \end{pmatrix} \right\|^2 \right\}$$

is decreasing

# Mathematical Model

(2-stage for simplicity)

$$\left\{\begin{array}{lll} \min & c^\top x + u + \frac{1}{1-\alpha}\mathbb{E}\left[v(\xi)\right] & \\ \text{s.t.} & x^t \in \mathcal{S}_{x^t}, \quad y^t(\xi) \in \mathcal{S}_{y^t}(\xi) & \text{for all } t, \text{ a.s. } \xi \\ & x_p \in \mathcal{D}_{x_p}, y_p(\xi) \in \mathcal{D}_{y_p}(\xi) & \text{for all } p, \text{ a.s. } \xi \\ & \left(x_p^t, y_p^t(\xi)\right) \in \mathcal{C}_p^t(\xi) & \text{for all } t, p, \text{ a.s. } \xi \\ & u \in \mathbb{R}, v(\xi) \geq 0 & \\ & v(\xi) \geq q(\xi)^\top y(\xi) - u & \text{for all } \xi \end{array}\right.$$

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CVaR coupling resolved by new PH variant

# Features preventing separability in the GEP

- **Spatial coupling:** **demand constraint**

  all generation assets at each time step: $\sum_p y_p^t(\xi) \geq d^t(\xi)$, for each $t$

- **Temporal coupling:** **ramps, water balance**

  one/few generation assets: $\left(y_p^1(\xi), \ldots, y_p^T(\xi)\right) \in \mathcal{D}_p(\xi)$, for each $p$

- **Stagewise coupling:** **capacity constraint**

  investment and generation decisions: $y_p^t(\xi) \leq cap_p + x$, for each $t, p$

- **Scenario coupling:** **induced by risk aversion**

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## Spatial coupling: variable splitting

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The bundle method requires minimizing subproblems below certain level, not until optimality (faster!)

**inexact proximal bundle method with noise attenuation**

# Handling inexact problem solution in PH

PH finds certain **saddle point**.

- $x^{k+1}(\cdot) = Proj_{\mathcal{N}}(\hat{x}^k(\cdot))$ $\Leftarrow$ min
- $w^{k+1}(\cdot) = w^k(\cdot) + rProj_{\mathcal{N}^\perp}(\hat{x}^k(\cdot))$ $\Leftarrow$ max

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- Augmented dual function is smooth, but only inexact information is available
- NSO guides the handling of such inexactness, by keeping memory of the iterative process (“bundle”)
- The approach is versatile and flexible: *inexactness* can be on-demand, solving more accurately some subproblems
- Theory gives optimality certificates

# Optimality certificates

In both primal and dual variables, up to the accuracy in the subproblem solution

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- Algorithm solves quadratic and linear programming problems at each iteration

# Numerical assessment on a 2-stage toy problem

- Consistent output, comparged to solving without decomposing
- Risk-neutral (left) and risk-averse (right), no decomposition

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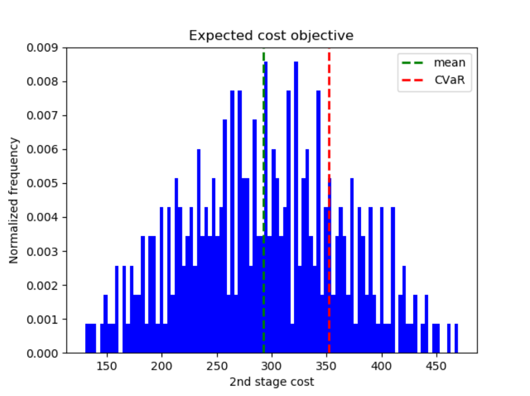


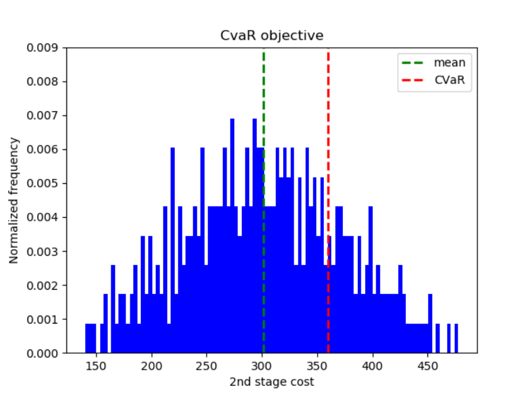

# Numerical assessment on a 2-stage toy problem

- Consistent output, comparged to solving without decomposing
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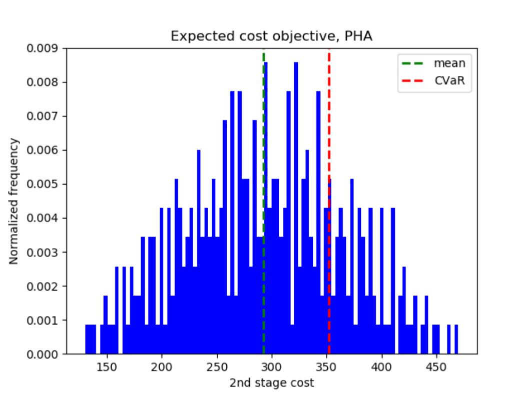


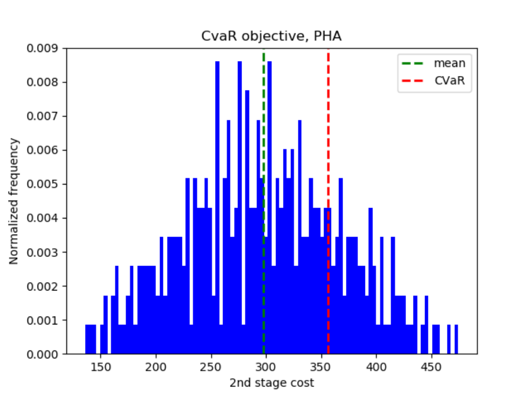

# Final remarks

**This is on-going work!** Points we need to explore

- Can we device a stopping test for PHA, à la bundle method

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- Final comment: PH with inexact subproblem solution applies in multistage setting, Benders not so much