Design of Liquid Crystals for Microscale Dynamics

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Outline

- Introduction: How LCs are different from water
- LC-enabled levitation
- Brownian motion in a LC
- Director distortions control placement of small particles
- LC-enabled electrokinetics
- Particle-like 3D solitons in LCs
- Dynamic topographies of LC elastomeric coatings
- Dynamics of swimming bacteria guided by LCs
Geometry of experiment on solitons

Planar cell, negative dielectric anisotropy, negative anisotropy of conductivity; at first sight, the electric field should not cause realignment as the director is already perpendicular to the field.

\[ \Delta \varepsilon = \varepsilon_\parallel - \varepsilon_\perp < 0 \quad \Delta \sigma = \sigma_\parallel - \sigma_\perp < 0 \]

CCN-47: (−,−) nematic
Director bullets as 3D solitons

Localized perturbations of the director; very different from a homogeneous tilt in pixels of LCDs

$E \circ \uparrow \hat{n}_0$

Bullets move perpendicularly to the initial director and to the electric field
Flexoelectric polarization as the cause of soliton formation

Director curvatures produce flexopolarization that couples to the electric field; resulting realigning torque and Coulomb force:

\[ \Gamma_\parallel = \mathbf{P}_\parallel \times \mathbf{E} \]

\[ F_\parallel \propto \rho_\parallel \mathbf{E} \propto (\nabla \cdot \mathbf{P}_\parallel) \mathbf{E} \]

Flexoelectric mechanism is supported by the polar director response: as the field polarity is reversed, the in-plane tilts preserve sign but the out-of-plane tilts change signs.

\[ \mathbf{P}_\parallel = e_1 \hat{n} \text{div}\hat{n} - e_3 \hat{n} \times \text{curl}\hat{n} \]
Flexoelectricity…and Activity

Flexoelectric polarization for dipolar molecules:

\[ \mathbf{P}_{\text{flex}} = e_1 \hat{n} \nabla \cdot \hat{n} - e_3 \hat{n} \times (\nabla \times \hat{n}) \]

Quadrupolar flexoelectricity:

\[ \mathbf{P}_{\text{flex}} = e \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \quad \mathbf{P}_{\text{flex},i} = e \partial_j n_i n_j \]

Active force in 3D

\[ \mathbf{f}_{\text{active},i} = \alpha \partial_j n_i n_j \quad \mathbf{f}_{\text{active}} = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

Spatially non-uniform tensor produces a vector (electric polarization or active force, depending on the physical context)

\[ \mathbf{f}_{\text{activation}} = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \quad \mathbf{f}_{\text{activation},i} = \alpha \partial_j n_i n_j \]

Director gradient-induced deformation of LC elastomers
Motivation to study LC elastomers:

Dick Broer invited me to visit his lab at the Eindhoven University of Technology, “Have a mini-sabbatical!”

After some contemplating, worries, emails “… Do you expect me to make elastomers in the lab? Are any acids involved? Flames? Should I bring my lab coat? Special eyewear? Would there be someone to teach me chemistry?”

Dick got a brilliant idea:

“Why don’t you bring a student with you?”

Greta Babakhanova
Just learned she will go to Eindhoven to learn how to make elastomers
LC Elastomers: Anisotropic rubber


![Image of elastomers in isotropic and nematic states with temperature-induced elongation/contraction diagram]

Temperature-induced elongation/contraction:


Figure 3. Typical chain backbones in the nematic and isotropic states, with their shape distribution being represented by the oblate and spherical shapes.
Motivation: Patterned LCE films

Director gradients allows one to control curvature in free films and surface topography in coatings…

Broer, Bunning, Ikeda, Liu, Palffy-Muhoray, Tabiryan, Terentjev, Verduzco, Ware, Warner, White…
Motivation: Epithelium

Can we create an artificial “skin” that transforms a 2D director field into a 3D topography with in-plane and out of plane response?

This work: Dynamic surface topography of LCE “skin” can be pre-programmed by patterned in-plane director field:

$$\hat{n}(x, y) \Rightarrow \delta x, \delta y, \delta z$$
Composition of LCE

a

RM82
(25wt%)

b

RM23
(25wt%)

c

RM105
(49.2wt%)

d

Irgacure 819
(0.8wt%)
Making liquid crystal elastomer coatings

1) Spin-coat photosensitive layer (azo-dye)

2) Create a sandwich-type cell

3) Photoalignment by irradiating the cell with light of spatially-varying linear polarization

4) Inject liquid crystalline monomeric mixture into the photoaligned cell

5) Photopolymerize the monomers at room temperature

6) Remove one of the glass substrates, the coating is supported by the second glass

Thickness: \( \sim 5 \ \mu m \)
Experimental examples of director patterns

Lattices of +1 and -1 defects

Bend +1

Splay +1

Experimental cell, not a numerical simulation; the ticks represent the director as seen under LC PolScope

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Bend: Elevations upon heating

Image by Digital Holographic Microscope, Lyncée tec, Frank Liu demonstrates the unit at ILCEC
Bend: Elevations upon heating

Image by Digital Holographic Microscope, Lyncée tec, Frank Liu demonstrates the unit at ILCEC
Bend: Elevations upon heating
Splay: Depletion on heating
Splay: Depletion on heating
Splay+Bend: Elevation/depletion and in-plane shifts
Splay+Bend: Elevation/depletion and in-plane shifts
Summary of experiment:

☐ In-plane director defines out-of-plane and in-plane displacements: \( \hat{n}(x, y) \Rightarrow \delta x, \delta y, \delta z \)
  - Bend causes elevations
  - Splay causes depressions
  - Bend-splay with dipolar symmetry around \( \frac{1}{2} \) disclinations cause shift toward the “tail” of the structure

☐ How to explain it?
  - Complex geometry
  - Different boundary conditions
  - Material moves…
Warner-Terentjev step length tensor: Heating effect

\[ l_{ij} = l_{\perp} \delta_{ij} + (l_{\parallel} - l_{\perp}) n_i n_j \]

\[ l_{\parallel} > l_{\perp} \]

\[ l_{\parallel}^{iso} = l_{\perp}^{iso} = \bar{l} \]
Warner-Terentjev step length tensor: Heating effect

\[ l_{ij} = l_\perp \delta_{ij} + (l_\parallel - l_\perp) n_i n_j \]

\[ l_\parallel > l_\perp \]

\[ l_{iso}^{\parallel} = l_{iso}^{\perp} = \bar{l} \]

Spatially varying pattern of ellipsoids (=director):

...produces activation force

Warner-Terentjev step length tensor: Heating effect

Spatially nonuniform tensor produces a vector \( f_i = \alpha \partial_j n_i n_j \)
which is a force with \( \alpha \sim F / \bar{T}^2 \sim \mu (l_\perp - l_\parallel) / \bar{T} \)

Invariant form: \( f = \alpha (\hat{n} \text{div} \hat{n} - \hat{n} \times \text{curl} \hat{n}) \)

\[ l_{ij} = l_\perp \delta_{ij} + (l_\parallel - l_\perp) n_i n_j \]
\[ l_\parallel > l_\perp \]
\[ l_\parallel^{\text{iso}} = l_\perp^{\text{iso}} = \bar{T} \]

\[ F \sim \mu \bar{T} (l_\perp - l_\parallel) \]

Bend: Elevations upon heating

\[ f = \alpha (\hat{n} \text{div}\hat{n} - \hat{n} \times \text{curl}\hat{n}) \]
Splay: Depletion on heating

\[ f = \alpha (\hat{n} \text{div} \hat{n} - \hat{n} \times \text{curl} \hat{n}) \]
Splay+Bend: Shift of $\frac{1}{2}$ disclinations towards their tails

\[ f = \alpha (\hat{n} \text{div} \hat{n} - \hat{n} \times \text{curl} \hat{n}) \]
Dynamic profiles to control placement of colloids at a photosensitive elastomer

G. Babakhanova et al, Submitted (2019)
Dynamic surface profile of the LCE coatings activated by temperature can be pre-programmed deterministically by inscribing in-plane director patterns. \( \hat{n}(x, y) \Rightarrow \delta x, \delta y, \delta z \)

Upon heating, bend causes elevations, splay causes depressions, their combinations trigger in-plane shifts (when the symmetry is right, as in the +1/2 case)

Activation forces are caused by stretching-contraction of the polymer network and spatially varying orientation of LCE; \( \mathbf{f} = \alpha (\nabla \cdot \mathbf{n} - \mathbf{n} \times \nabla \times \mathbf{n}) \)

Rather unexpected connection to flexoelectricity and active matter through the activation force

Can be used for placement of colloids
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Bacteria: Untapped reservoir of mechanical energy

Effective swimmers at Re<<1, bacteria are “ready-to-use” for microtransport, delivery, mixing

Howard C. Berg, Harvard U: *E. Coli*

Prokariotic microorganisms, few micrometers in size
One of the earliest forms of life on Earth
Total number: 5 x 10^{30}; bacterial biomass on Earth larger than that of all plants and animals combined
Our body has more bacterial cells than human cells
Can we force bacteria to perform a useful work for us? To deliver cargos, drugs, to run micromachines, etc? Can we “domesticate” swimming bacteria?
Domestication: by visual, auditory, and tactile communications

Humans learned how to control animals in terms of their concentration…

…trajectories of motion…

…and polarity of motion…
Can we control dynamics of micro-organisms?

Challenge: Communication channels are few and ineffective:

1. Most micro-organisms are blind and deaf; “touching” might be detrimental
2. Gradients of nutrients/chemicals: bias in bacterial concentration and trajectories, but only temporarily, once the gradients diffuse, the control is lost

This work:
Use liquid crystals as communication channels to command swimming bacteria
Navier-Stokes Equations and The Scallop Theorem


\[ \text{Re} = \frac{\text{inertia term}}{\text{viscosity term}} = \frac{\rho VL}{\eta} << 1 \quad \Rightarrow \quad \eta \nabla^2 v = \nabla P \]

At low Reynolds number, there is no time dependence; the strokes must be non-invariant under time reversal to allow swimming.

Scallop cannot swim at low Re

Corkscrew and flagellated bacteria can swim at low Re
Mechanism of filament propulsion in water

Projection of the drag force onto the x-axis creates a propulsive force when the two friction coefficients are different; for a thin cylinder, \( \xi_\perp \approx 2 \xi_\parallel \)

Simultaneous reversal of normal displacement and of tilt preserves the sign of propulsive force, thus enabling forward motion

\[ v \rightarrow -v; \; \theta \rightarrow \pi - \theta \Rightarrow \mathbf{f}_{\text{prop}} = \text{const} \]

However, back and forth displacement produces no thrust: \( v \rightarrow -v; \; \theta \rightarrow \theta \Rightarrow \langle \mathbf{f}_{\text{prop}} \rangle = 0 \)

\[ \nu_{\text{bacterium}} \propto \theta \frac{\xi_\perp - \xi_\parallel}{\xi_\parallel} \text{ frequency} \]

Lauga, Powers, Rep Prog Phys 72, 096601 (2009)
Force-free swimming: Pusher type, the axial flow is away from the bacterium; the equatorial flow is towards the bacterium

Hydrodynamic force dipole $\sigma = F_{prop} l \approx -(10^{-17} - 10^{-18})$ N m


Activity: $\alpha = \sigma c = F_{prop} l c \approx -(10^{-3} - 10^{-1})$ N / m$^2$

for $c \sim (10^{15} - 10^{16})$/m$^3$
Which liquid crystal to place the bacteria in?

Thermotropic liquid crystal from TVs and laptops? No, this is an oily hydrophobic fluid

Lyotropic liquid crystal formed by surfactants in water? No, this is a soap, kills bacteria

Correct answer: Lyotropic chromonic liquid crystals (LCLCs, or chromonics), water dispersions of polyaromatic organic molecules.
Chromonic LCs: non-toxic

- Chromonic molecules have no aliphatic tails, which makes them friendly to biological systems.

Sunset Yellow, aka Yellow #6, food colorant

Approved by US FDA for intake by humans:

This work: DSCG, a.k.a. cromolyn, anti-asthma drug, concentration 14-17 wt%, friendly to bacteria and their nutrients (Terrific Broth).
Lyotropic Liquid Crystals: Power of Entropy

Onsager (1949): Nematic order in solution of long thin rods, thanks to translational vs orientational entropy trade-off

Concentration, c
Lyotropic Liquid Crystals: Power of Entropy

Onsager (1949): Nematic order in solution of long thin rods, thanks to translational vs orientational entropy trade-off

Concentration, $c$

Tobacco mosaic virus (TMV)

J. Bernal and I. Fankuchen, J. Gen. Physiol. (1941): tactoids as N nuclei in tobacco mosaic virus dispersions
Molecules have no aliphatic tails, thus chromonics are friendly to biological systems.

Chromonic aggregates resemble DNA with that difference that there are no covalent bonds between units; “sticking” energy is only $\sim 10 k_B T$; easy to assemble and disassemble.

Sunset Yellow, aka Yellow #6, food colorant; Alura Red, Tartrazine, many others chromonics are also food colorants.

Approved by FDA for intake by humans:
Lyotropic Chromonic LCs: special case

1D aggregation in dilute (not nematic) solution of neutral molecules: balance of entropy and association energy

\[ F = \sum_L c(L) \delta + k_B T \sum_L c(L) \ln \left[ c(L) v_0 \right] \]

Entropy of mixing

Minimizing the free energy over \( c(L) \)
under the constraint of a constant volume fraction
\[ \sum_L L c(L) v_0 = \frac{v_0}{L_0} \int_0^\infty dL L c(L) = L_0 \phi = \text{const} \]
yields

\[ c(L) = \exp \left(-\frac{\delta}{k_B T}\right) \exp \left(-\frac{L}{\bar{L}}\right) / ev_0 \]

\[ \bar{L} \propto \sqrt{\phi} \exp \frac{\delta}{2k_B T} \]

The most probable aggregate length

Dual character of (LC)^2: T and c controlled

Onsager systems:
same rods, athermal, aligned at high c

Thermotropic LC (in your displays): same molecules, \( c = \text{const} \), aligned at low \( T \)

Chromonics:
controlled by both \( c \) and \( T \)

The average length in isotropic solution:
\[
\bar{L} \propto c^{1/2} \exp \frac{\delta}{2k_BT}
\]


\[
\bar{L} = L_0 c^{5/6} \left( \frac{\lambda_p}{D} \right)^{1/3} \exp \frac{\delta + \kappa \phi}{2k_BT}
\]

The field for at-depth theoretical exploration of the phase diagrams and viscoelasticity is wide open
Typical phase diagram: Sunset Yellow

Sunset Yellow
a.k.a Edicol
Food dye

H.-S. Park and O.D. Lavrentovich, Lyotropic Chromonic Liquid Crystals: Emerging Applications, Chapter 14, pages 449-484, in: Liquid Crystals Beyond Displays, Edited by Quan Li, Wiley, 2012

M. Taylor, J. Herzfeld, PRA 43, 1892 (1991)
Biphasic regions: Toroids and Tactoids

Nematic nuclei in the isotropic phase; note structural chirality

- Surface energy  \( F_S \sim \sigma \parallel Rr \sim \sigma \parallel \sqrt{RV} \)
- Elastic energy  \( F_B \sim KV / R^2 \)

\[
\frac{R^2}{r} \approx \beta \frac{K}{\sigma} \]

Tortora, ODL, PNAS (2011)
Why Chromonics?

The term Chromonic was introduced by John Lydon who wrote a number of reviews on LCLCs, the latest Liq. Cryst. 38, 1663 (2011); see also P. Collings et al., Mol.Cryst.Liq.Cryst. 509, 9 (2009)

- To carry connotations of dyes, chromosomes, and Cromolyn, the most studied (as an anti-asthma drug) material
- Shorthand for a lyotropic mesophase formed by soluble aromatic mesogens

Cromolyn or disodium cromoglycate (DSCG), or Intal; anti-asthma drug
Liquid Crystals + Bacteria = Living Liquid Crystals

**Active matter** with an independent control of orientational order and activity

*B. Subtilis*, swims by rotating helical flagella; activity is controlled by oxygen/nitrogen supply

Body: 2-5 \( \mu m \)

Chromonic LC

Water based LC, non-toxic for bacteria

Content

- Bacteria in uniform LC
  - Individual behavior
  - Collective instabilities

- Bacteria in non-uniform (patterned) LC
  - Command of trajectories and concentration distribution by director gradients
Uniform alignment, low $c_B$: bacteria follow $\hat{n}$

$c_B < 10^{14} / \text{m}^3$

Video rate $\frac{1}{4}$

Swimming parallel to the director

Swimming by rotating flagella

Flagella rotation: 16 Hz
Body rotation: $f=2.5$ Hz

Velocity is proportional to frequency of flagellum rotation

\[ v_{\text{bacterium}} \propto \frac{\xi_\perp - \xi_\parallel}{\xi_\parallel} \text{frequency} \]

Bacteria swim faster than naively expected

Speeds in LCLC and pure water are comparable. Propulsion depends on viscous anisotropy rather than the absolute viscosity

\[
\nu_{\text{bacterium}} \propto \frac{\xi_\perp - \xi_\parallel}{\xi_\parallel} \text{frequency}
\]

Swimming along the director: cargo transport

Bacterium swims parallel to the director, like a train; if there is an obstacle, bacterium pushes it forward; this mode of "cargo transport" is impossible in isotropic fluids.

Why bacterium swims parallel to director

Bacterium’s realigning torque:

\[ \tau_{\text{bacterium}} \sim \alpha F_{\text{prop}}l \sim \alpha \times 10^{-17} \text{ N m} \]

Stabilizing elastic torque for infinitely strong anchoring:

\[ \tau \sim \alpha Kl \sim \alpha \times 10^{-16} \text{ N m} > \tau_{\text{bacterium}} \]

For finite anchoring, the stabilizing torque is determined by the surface properties; it is weak, thus allowing the bacteria to realign perpendicularly to the director when it is hungry

\[ \tau \sim \alpha Wlr \sim \alpha \times 10^{-18} \text{ N m} < \tau_{\text{bacterium}} \]

\[ W = 10^{-6} \text{ Jm}^{-2}; r = 0.5 \mu\text{m} \]

Swimming perpendicularly to the director:

$d=5\text{-}25\mu m$

Homeotropic cell

1. Tilt
2. Realignment; Different symmetry of distortions

Swimming perpendicularly to the director and tumbling

Howard C. Berg,
*E. Coli* in water:
run-and-tumble in search for nutrients

Backtracking as tumbling scenarios in LCLC
Swimming perpendicularly to the director and chaining

Director relaxation rate is finite; Corridors of distortions attract other bacteria
Content

☐ Individual bacteria in LC:
  ■ Swimming along the director; fast; cargo transport

☐ Collective effects in uniformly aligned living LC

☐ Collective effects in a LC with a distorted director
Activity increase (velocity $v$ or concentration $c_B$) causes a transition from uniform equilibrium to bend instability

Bacteria are inactive (no oxygen)

$c_B \sim 10^{15} \text{ /m}^3$

- Equilibrium nematic,
- Velocity=0,
- $n(\mathbf{r}) = \text{constant}$

Bacteria are active (added oxygen)

- Out-of equilibrium
- Velocity $> 0$
- $n(\mathbf{r}) = \text{periodic bend}$

Activity increase (velocity $v$ or concentration $c_B$) causes a transition from uniform equilibrium to bend instability.
Activity increase (velocity $v$ or concentration $c_B$) causes a transition from uniform equilibrium to bend instability.

As activity (=concentration) increases, the bacteria create bend instabilities.
Equilibrium nematic vs active nematic:
What is the mechanism of different behavior?

Equilibrium nematic:
Elasticity quenches fluctuative bend

Active nematic:
Active forces enhance fluctuative bend

New term in Navier-Stokes equations:
\[
\rho \frac{Dv_k}{Dt} = -\partial_k P + \eta \nabla^2 v_k + \alpha \nabla \cdot \left( n_j n_k \right) + \partial_j \left( \lambda_{ijk} \frac{\delta F_{\text{FrankOseen}}}{\delta n_i} \right)
\]


Invariant form of destabilizing active force:
\[
f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times \left( \nabla \times \hat{n} \right) \right]
\]

\( \alpha \propto c F_{\text{prop}} l \)
Balance of elasticity vs activity defines bending period

Torque by bacteria generated flow

\[ \sim \alpha \theta \sim c F_{prop} l \theta \]

Stabilizing LC elastic torque

\[ \sim K \frac{\partial^2 \theta}{\partial x^2} \]

Periodicity:

\[ \xi \sim \sqrt{\frac{K}{\alpha}} \]

Activity vs Flexoelectricity

Flexoelectric polarization for dipolar molecules:

\[ P_{\text{flex}} = e_1 \hat{n} \nabla \cdot \hat{n} - e_3 \hat{n} \times (\nabla \times \hat{n}) \]

Quadrupolar flexoelectricity:

\[ P_{\text{flex}} = e \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

flexocoeficient

\[ f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

activity
Even higher activity: Walls replaced by disclinations

As activity increases, the uniform state (1) undulates, then (2) nucleates disclination pairs. Similar 2 stage scenario is seen in numerical simulations of active matter: Thampi et al, EPL (2014); Shi, Ma, Nature Comm (2013)

Director within the pair realigned by 90° w.r.t. the original director

Two-step development of topological turbulence

Equilibrium nematic

Increase of activity (oxygen concentration)

Active turbulent nematic

Walls: \( F_w \propto \frac{K}{\xi r_{\text{wall}}} \)

Defect pairs: \( F_{\pm 1/2} \propto \frac{K}{\xi_d^2} \ln \frac{\xi_d}{r_c} \)

\[
F_{\pm 1/2} / F_{\text{wall}} = \xi r_{\text{wall}} \ln \left( \frac{\xi_d}{r_c} \right) / \xi_d^2 < 1
\]

Increasing cost of bend leads to nucleation of +/- ½ defects

Content

- Bacteria in uniform LC:
  - Individual behavior: Swimming along the director or realigning the director when hungry; cargo transport
  - Two-step transition from uniform equilibrium to topological turbulence

- Bacteria in non-uniform (patterned) LC
Alignment of living liquid crystals

Discussed so far:
Uniform alignment

Rest of the talk: Patterned photo-alignment, which imposes external activity:

\[ f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \neq 0 \]

Green, Toner, Vitelli, PR Fluids 2, 104201 (2017)
Pure splay: Bacteria swim \parallel director; bipolar motion

\[ splay = \hat{n} \nabla \cdot \hat{n} \neq 0 \]

Number of bacteria swimming towards and away from the core is about the same at low \( c_B \)

\( c_B = 10^{14} \text{ /m}^3 \)
Pure splay: Bacteria swim \parallel \text{ director; bipolar motion}

\[ splay = \hat{n} \nabla \cdot \hat{n} \neq 0 \]

\[ c_B = 10^{14} \text{ /m}^3 \]

Number of bacteria swimming towards and away from the core is about the same at low \( c_B \)

High \( c_B \): Some bacteria are trapped at the defect cores
Pure bend: Bacteria swim || director; bipolar motion

Number of bacteria swimming to the right and to the left is the same, as expected

\[ c_B = 10^{14} / \text{m}^3 \]
Mixed splay-bend: counterclockwise polar motion
Mixed splay-bend: counterclockwise polar motion

c_B = 10^{14} /m^3
Mixed splay-bend: counterclockwise polar motion

\[ c_B = 10^{14} \text{ /m}^3 \]

Spiral vortex:
Counterclockwise collective circular swimming; strictly polar

Periodic pattern of -1 and +1 defects: Bacteria gather and circulate around each +1, avoid -1

Periodic pattern of -1 and +1 defects: Bacteria gather and circulate around each +1, avoid -1

Mechanisms?

Active force

\[ \mathbf{f} = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

Spatially non-uniform tensor produces a vector ("active force")

\[ f_i = \alpha \partial_j n_i n_j \]

\[ \mathbf{f}_{\text{splay}} = \alpha \left[ \hat{n} \nabla \cdot \hat{n} \right] \]

\[ \mathbf{f}_{\text{bend}} = \alpha \left[ -\hat{n} \times (\nabla \times \hat{n}) \right] \]
Mechanisms?

Active force \[ \mathbf{f} = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

Spatially non-uniform tensor produces a vector ("active force")

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Mechanisms?

Active force

\[ \mathbf{f} = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

\[ \hat{n} = (n_x, n_y, n_z) = (\cos \theta, \sin \theta, 0) \]

\[ \theta(x, y) = \tan^{-1} \frac{y}{x} + \frac{\pi}{4} \]

\[ \mathbf{f} = \left\{ 0, -\frac{\alpha}{r} \right\} \] azimuthal force

active splay  +  active bend  =  \[ \mathbf{f} = \left\{ 0, -\frac{\alpha}{r} \right\} \]
Mechanisms?

Active force

\[ f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \nabla \times (\nabla \times \hat{n}) \right] \]

\( \hat{n} = (n_x, n_y, n_z) = (\cos \theta, \sin \theta, 0) \)

\[ \theta(x,y) = \tan^{-1} \frac{y}{x} + \frac{\pi}{4} \]

\[ f = \{0, -\alpha / r\} \quad \text{azimuthal force} \]
Mixed splay-bend: Forces unipolar circulation (as opposed to bipolar)

\[ f = \{0, -\alpha / r\} \]

\[ f_{\text{drag}} = \eta \nabla^2 v \]

Opposing viscous force

Balance of drive and drag:

\[ v = \begin{cases} 
0, & \alpha r \log \left( \frac{r}{r_0} \right) \\
\frac{\alpha r}{2\eta} & \end{cases} \]

Polar coordinates

Deduce \( \alpha / \eta \approx -0.7 \text{ s}^{-1} \)

Active force deflected by -1 defects

\[ f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

the active force deflects away from the center

-1 defect director
Linear pumping: from +1/2 defect to -1/2 defect

\[ f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \]

Linear pumping: from $+1/2$ defect to $-1/2$ defect

Bacteria concentrate at the cores of $+1/2$ defects; experiment without patterned director

Unipolar swimming in concentrated jets:

Unipolar transport

As the jets undulate, the angle between the director and the trajectory increases; viscous drag increases and some bacteria are ejected; it helps to keep the period and amplitude of undulations constant; The jets are stable at \( c < 7c_{\text{critical}} \) where \( c_{\text{critical}} \) is the critical concentration of instability in a uniform cell.
Unipolar swimming in concentrated jets:

Isotropic phase; few degrees higher temperature

Unipolar transport: When the experiment and common sense are at odds with the math:

A simple description with active force \( f = \alpha \left[ \hat{n} \nabla \cdot \hat{n} - \hat{n} \times (\nabla \times \hat{n}) \right] \)
does not work, since it predicts both left and right motion:

Binary character of the living liquid crystal and concentration variations makes the standard model incomplete:

\( c_B \neq \text{const} \)
LCs command swimming bacteria

- Bacteria in uniform LC:
  - Director controls trajectories of swimming; cargo transport
  - Two-step transition from equilibrium aligned state to topological turbulence as activity increases above the elastic threshold

- Bacteria in patterned LC:
  - Patterns control trajectories, polarity and concentration of bacteria;
  - Bacteria recognize topological charges, concentrate at positive and avoid negative ones
  - Patterns cause unipolar threshold-less flow of bacteria, circular or linear
Outline

- Introduction: How LCs are different from water
- LC-enabled levitation
- Brownian motion in a LC
- Director distortions control placement of small particles
- LC-enabled electrokinetics
- Particle-like 3D solitons in LCs
- Dynamic topographies of LC elastomeric coatings
- Dynamics of swimming bacteria guided by LCs