Isaac Newton Institute for Mathematical Sciences

Case study

A Quantum Leap at INI

Case study: Mathematics for Quantum Information

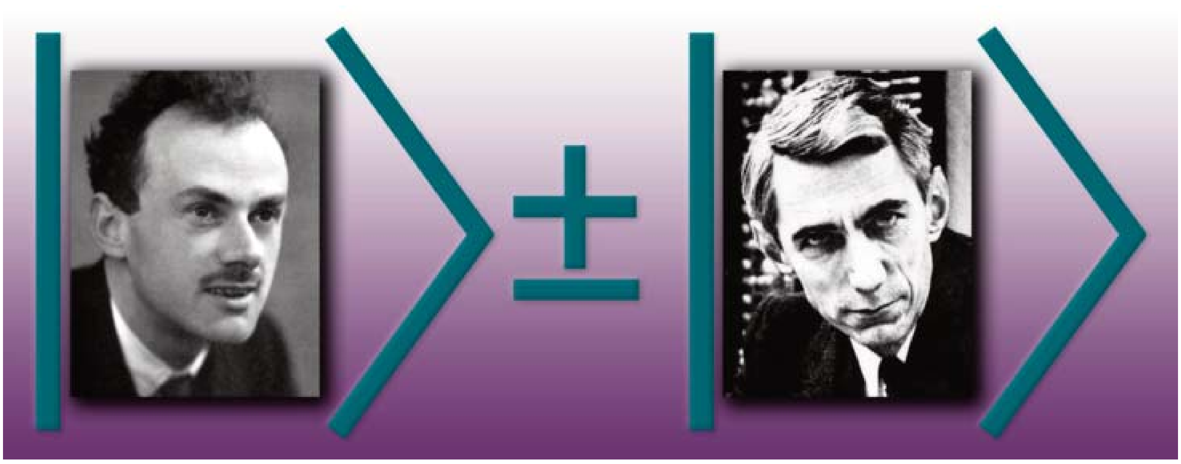
"The application of quantum technologies to encryption algorithms threatens to dramatically impact the US government's ability to both protect its communications and eavesdrop on the communications of foreign governments," according to an internal document leaked by whistle-blower Edward Snowden. That the UK Government takes quantum computing seriously is evinced by its commitment to "provide £270 million over 5 years to fund a programme to support translation of the UK's world leading quantum research into application and new industries – from quantum computation to secure communication", as announced by Chancellor of the Exchequer George Osborne in his 2013 Autumn Statement. This statement by the UK Chancellor came during the closing weeks of an INI programme on *Mathematics for Quantum Information* (MQI) and within days of an announcement by INI participants of a major breakthrough in their field.......

The excitement rippling around the building when Garcia-Patron presented this proof in the final week of the programme was palpable.

In their original proposal, two years earlier, MQI programme organisers Richard Jozsa (Cambridge), Noah Linden (Bristol), Peter Shor (MIT) and Andreas Winter (Bristol/CQT, Singapore) had correctly anticipated the readiness of their discipline for such breakthroughs predicting

> "*we see this as a moment of substantial scientific opportunity: we believe we are at the beginning of a period in the subject where diverse areas of mathematics will play an increasing important role. A Newton Institute programme at this time offers the possibility of major influence in the speed and direction of the field*".

In October 2013, Alexander Holevo, a pioneer in the field of Quantum Information Science and winner of the Humbolt Prize, gave the Rothschild Lecture for the MQI programme. Holevo, Professor at Moscow State University and the Moscow Institute of Physics and Technology, and a member of the Steklov Mathematical Institute, closed his talk on Quantum Shannon Theory with a challenge for his fellow participants as he outlined two long-standing open problems in the field: *the existence of optimal Gaussian inputs for Gaussian channels* and *the additivity of Gaussian entropic quantities*.

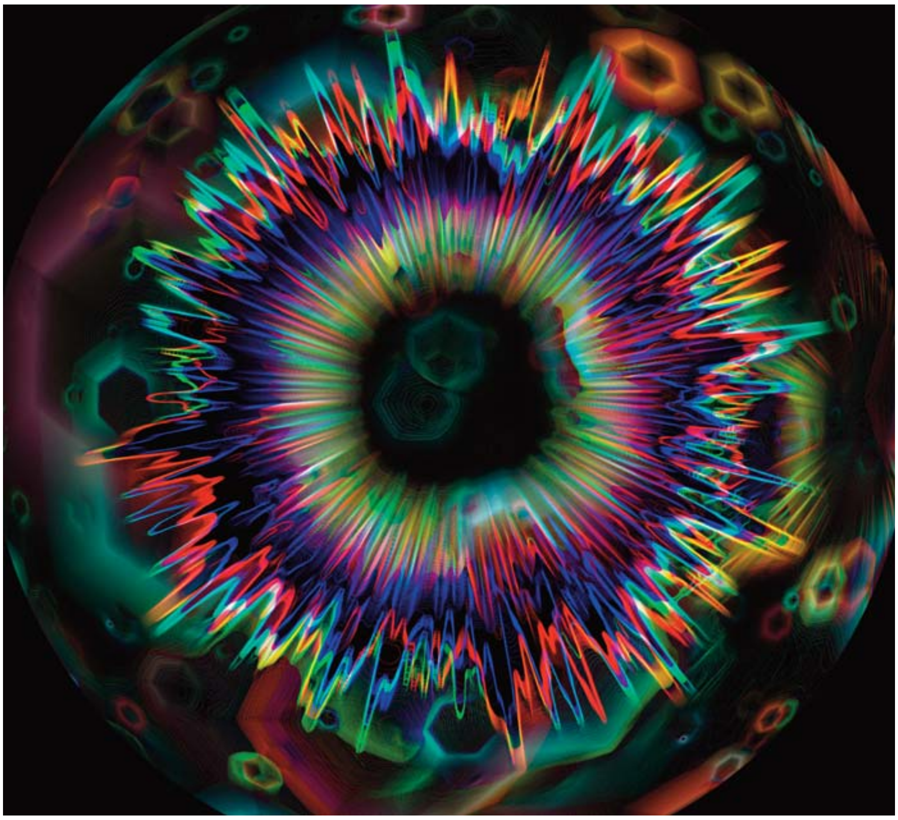


Paul Dirac and Claude Shannon depicted on the programme logo

Less than three months later, following hours gathered around the Newton Institute's famous chalkboards, Holevo and two other participants on the MQI programme, Vittorio Giovannetti (Scuola Normale Superiore, Pisa, Italy) and Raul García-Patrón (Ecole Polytechnique de Bruxelles, Belgium, and Max-Planck Institut für Quantenoptik, Germany), announced that they had solved these long-standing conjectures. The excitement rippling around the building when García-Patrón presented this proof in the final week of the programme was palpable.

The field of Information Theory was created in a single stroke by Claude Shannon in 1948. Classical Shannon Information Theory and its quantum counterpart both concern the communication of information along channels: messages are encoded, transmitted, received and then decoded. Examples of channels include telephone lines through which electricity can travel and fibre optic cables through which light can travel. Quantum channels differ from their classical counterparts in that they can transmit quantum information in terms of qubits: within the classical framework a bit has two possible states, 0 and 1, whereas within the quantum framework a qubit can also be the superposition of these states (recall Schrödinger's cat – until observed it can be both alive and dead.)

Holevo, Giovannetti and Garcia-Patron acknowledge "the catalyzing role of the Isaac Newton Institute" in their ending the 40-year quest for the capacity of Bosonic quantum channels.



Naturally, those transmitting and receiving messages wish to maximize the rate at which information can be communicated whilst preserving the faithfulness of the output to its source. The issue is that "noise" along these channels often results in errors, or discrepancies between the output message and the original input. Thus a vital measure is the capacity of a channel, defined as the maximum rate at which information can reliably be communicated along the channel. In his 1948 theory, Shannon considered classical channels with additive Gaussian noise; that is, the received signal is the sum of the transmitted signal and noise which follows a Gaussian, or normal, distribution. For such channels he derived a formula for the capacity of the channel in terms of its signal to noise ratio.

Holevo, Giovannetti and García-Patrón's achievement during their time at INI was to obtain and prove the correctness of quantum counterparts to the Shannon formula. They proved that for a broad class of Bosonic Gaussian channels, the capacity is optimised when the transmitted signal is in the vacuum state, that is the quantum state with the lowest possible energy. Ironically, but as has happened in the proof of countless mathematical theorems, this breakthrough came about as a direct result of a push by these individuals, along with fellow programme participant Reinhart Werner (Universität Hannover) to find a counter example to the problem.

Holevo, Giovannetti and García-Patrón acknowledge "the catalyzing role of the Isaac Newton Institute" in their ending the 40-year quest for the capacity of Bosonic quantum channels.[1] Programme Organiser Peter Shor, Professor of Applied Mathematics at MIT, said: "*The solution of this problem is, within our field, like Wiles' solution of Fermat's Last Theorem*", another long-standing conjecture for which a proof was presented at INI.

References

[1] Giovannetti, V., Holevo, A.S., and García-Patrón, R. (2014) 'A solution of Gaussian optimizer conjecture for quantum channels', *Commun. Math Phys.* (DOI) 10.1007/s00220-014-2150-6.

MQI programme information is at **www.newton.ac.uk/event/MQI**

Isaac Newton Institute for Mathematical Sciences
20 Clarkson Road, Cambridge, CB3 0EH, UK
tel: 01223 335999 **email:** info@newton.ac.uk
web: www.newton.ac.uk