The Earth’s climate system is complex with a multitude of spatial and temporal scales. Although governing equations of many aspects are well understood, others are not easily represented in computer models (“simulators”) and it is not feasible to represent smaller-scale processes in a global climate model (GCM). Transfers between scales are known to be important and often may be best regarded as stochastic at resolved and unresolved scales. Thus uncertainties of GCM formulation lead to uncertainties in projections of future climate, as do uncertainties in initial conditions and forcing scenarios (e.g. volcanic aerosols and especially greenhouse gases). Yet societal need for good climate projections and a proper assessment of their uncertainties is pressing.

The programme ran from 11 August to 22 December 2010. It had two main themes:

- A: Stochastic parameterisation schemes and statistical implications;
- B: Probabilistic climate prediction.

The programme was unprecedented in bringing together more than 100 leading experts from mathematics/statistics and climate science. Several experts participated for a majority of the programme duration.

Some of the exciting problems tackled were:

- Development of stochastic models to represent, at scales simulated in a GCM, the mean and variability of effects of unresolved processes, to improve validity of climate simulations;
- Relationships between (i) emulators (statistical representations of complex models, used to explore dependencies on uncertain parameter values), (ii) simpler or coarser-resolution simulators, (iii) application of Maximum Entropy Production and (iv) GCMs (having the finest resolution among i-iv);
- Development and evaluation of statistical frameworks needed to create credible and reliable probability distributions of real-world observables from multi-model ensembles of climate projections (varied models, parameters, scenarios, initial and forcing conditions). Such frameworks are required in order to make valid inference from multi-model climate projections containing model error (biases).

Structure
Three workshops and an “Open for business” event were organized as follows.

The theme A workshop *Stochastic methods in climate modelling* at the Institute, 23-27 August, corresponded approximately with the first main theme. The presentations and discussions revealed several insights regarding the use of simple (‘phenomenological’) models, a highly relevant topic given recent advances in statistics that allow these models to be fitted to measurements, making due allowances for their limitations. There is an exciting opportunity for the expertise being built up in this area to transfer to stochastic schemes for sub-grid-scale processes.

The theme B workshop *Probabilistic Climate Prediction*, 21-23 September at the University of Exeter, was deliberately interactive; breakout and plenary discussion sessions equalled the time of presentations. Ten emerging research questions and discussion were placed on a Wiki for all programme participants to follow up. These questions are summarised here as four challenges:
1. Develop statistical frameworks for multi-model ensembles (MMEs) that can be used to make reliable predictions of societally-relevant variables. Ideally the frameworks will
   • incorporate process information from models (e.g. include physical metrics)
   • be able to cope with complex biases (e.g. non-linear, time-dependent etc)
   • inform about Design of Experiments for future MMEs that include climate models with different complexity and resolution
2. Develop rigorous procedures for evaluating probabilistic climate predictions and the assumptions used to construct them;
3. Find better ways of communicating the various sources of uncertainty in probabilistic climate predictions (e.g. dynamic graphics, best judgements).
4. Reconcile operational and conceptual probabilistic definitions of “climate” and “climate trend”.

Climate Change Question Time at Willis, London on 24th November attracted about 250 delegates from a wide range of interests in climate. A keynote presentation by Tim Palmer, the programme’s Rothschild Visiting Professor, was followed by panel discussions: The scientific uncertainties and their implications; Policy in the face of the uncertainties. Eminent panel members responded strongly to questions posed and the event was judged very successful.

A near-end workshop Uncertainty in climate prediction: models, methods and decision support, 6-10 December, combined the two main themes and included some emphasis on user needs. There were presentations describing progress in the programme, including the Rothschild Lecture, some presentations by scientists not otherwise in the programme, and “stakeholder” contributions especially regarding user needs of climate science. Several discussion sessions identified (i) programme achievements, (ii) challenges for climate science, (iii) how to disseminate programme outputs and sustain engagement with users, (iv) future directions and how to enable them.

Intervening periods of the programme were not formally structured but focused discussions included:
   • Emulators and “particle filtering” (September 2)
   • Palaeo-climate reconstruction and SUPRAnet (September 15)
   • Substantial presence at the CRASSH meeting "Challenging Models in the Face of Uncertainty" (September 28-30), including a debate on the future of climate modelling (between Jonty Rougier and Tim Palmer).
   • Tipping points (October 18-21, followed by an allied Royal Statistical Society meeting “Complexity and Statistics: Tipping Points and Crashes”, October 22, London)
   • Multi-model ensembles / probabilistic climate projection (October 26)
   • Substantial presence at “Extreme Environmental Events” (December 13-17; European Science Foundation), in Cambridge partly on account of the “Climate” programme.

Apart from the workshops, there were 49 individual talks; participants were encouraged to give these early in their visit to foster interaction in the programme.

Outcome and achievements
The programme brought together scientists working with observed climate-related data, idealized mathematical models, complex GCMs / simulators, and statisticians. Different needs and concerns of the modelling and user communities were recognised. This led to (i) a common understanding of need for a joint effort of statisticians and climate modellers to form climate projections stating uncertainties and usable by stakeholders, (ii) better agreement over concepts and thus (iii) a common sense of direction. Aspects of these developments are
- Acceptance and a raised profile of modelling uncertainty in climate science, the need to test models, ideas to deal with uncertainty, tightening vocabulary, clarifying issues.
- Impetus to think about fundamentals of climate modelling and combining state-of-the-art statistics and modelling (projects instigated).
- A boost to, and sound statistical framework for, paleoclimate and tipping point studies: dynamical models, use of proxies, connections between future and palaeo methodology. We see the combination of state-of-the-art statistics and modelling as the way forward for climate science and its use for policy and investment decisions. The programme stimulated discussion about issues which will influence the writing, and potentially the conclusions, of the IPCC AR5 report.

The following describes particular advances made during the programme, and some new collaborations. Final reports indicate two to five new collaborations per participant.

**Framework for handling uncertainty.** Developments in analysis and interpretation of ensemble experiments included the weighting of simulators, handling deficiencies, decision-relevant probabilities. Other aspects discussed included phenomenological models for studying complex systems, data assimilation to narrow down parameter distributions, approximations (e.g. with emulators) for large problems, Bayesian methods to reconstruct palaeoclimates. Progress was made on extending ensemble-based data assimilation towards non-Gaussian schemes. Initial studies have been performed on Gaussian mixture models. NCAR and UCLA / ENS (new collaboration) are looking at a novel hybrid between ensemble and variational data assimilation, using strengths of each to estimate an optimal initial state for predictions.

For climate studies, one can consider an equation (e.g. Fokker-Planck) for the probability distribution (pdf) of all variables. The state of the climate system is a pdf. Observations provide information about the pdf and assimilation should lead to a best estimate. Assimilation normally involves an ensemble of solutions, i.e. for climate, an ensemble of pdfs, each with a certain probability. This “pdf of pdfs” should be governed by something like an infinite-dimensional Fokker-Planck equation. There are plans to pursue this approach further.

**Probabilistic Projections.** The “ASK” method was debated; it applies scaling factors (based on “detection and attribution”) to simulator output to adjust space-time patterns associated with different forcings; trends are then projected with associated uncertainties. Development will continue. It was agreed that any interpretation of climate projections needs to make clear any methods and assumptions.

Observation-based stochastic models using Linear Inverse Modelling (LIM) have shown that the trend in the most energetic sea-surface-temperature pattern is improbable on the basis of natural variability alone. NOAA, Univ. Oxford and Univ. Colorado will collaborate to evaluate trends relative to internal variability in IPCC models using LIM-based techniques. LIM can provide information on patterns of variability and “forcing”, and may also be applied for MJO and ENSO behaviour of the MetOffice seasonal coupled GCM, as possible diagnostics for climate model performance.

Optimal trade-off for finite computing has been analysed: ensemble (spanning key sources of uncertainty to assess uncertainty, risk or extreme events) versus refinement (improving simulator parameterisations and/or spatial resolution).

**Emulators.** An emulator is first fitted (with diagnostic checks) to a designed set of simulator runs, then used for statistical inference on the simulator outputs. A “toy” example has shown emulator utility even for very non-linear functions (paper submitted). Emulation of extremes is being investigated, allowing for uncertainty from error in the model inputs. The role of experimental design has been developed. Bayesian neural net emulators have been extended to account for uncertainty within a large-scale calibration exercise. Multivariate emulation of HadCM3 and other output has been demonstrated, including use of emulators to learn about simulators (structure). Emulator-based palaeo-climate reconstructions, synthesising GCM
evaluations and proxy reconstructions, and including a structured assessment of uncertainty, were presented by programme participants at the 2010 AGU Fall Meeting.

Paleo-climate and proxy data. Reconstructing past climates (maps) from proxy data is a complex statistical problem; the climate and proxies are multivariate and observations are sparse. A chronological scale is wanted, but each proxy type responds to climate distinctively and translates time to a depth scale differently. A paper was written on statistical methods for chronology building in palaeoclimate reconstructions. Discussion led to recommendations including: use of state-of-the-art statistical methods, transparency on modelling choices and uncertainties, expression of model discrepancy, reality as a ‘hidden’ state vector. Palaeoclimate scientists need to be appraised on dynamical systems and statistical inference. Members of SUPRAnet (Studying Uncertainty in Palaeoclimate Reconstruction; comprising statisticians, climate modellers and experts on proxies; Leverhulme Trust funding) met within the INI Programme. Synchronisation of idealised palaeoclimate models with the “astronomical forcing” was studied (paper submitted).

Tipping points. $\delta^{18}O$ in ice cores shows variability changing to larger amplitude and lower frequency at the end of the ice age. A lag-1 autocorrelation function may be an indicator for destabilisation of the cold glaciated northern hemisphere state. Varying numbers of climate states during the last ice age have been inferred. Indicators via variability have been analysed temporally and spatially, devising algorithms for the more novel spatial aspect (for GCMs).

A theoretical model of rainfall, related land cover and terrestrial carbon was developed to investigate Amazon forest die-back. Rainfall can be reduced, until feedbacks no longer maintain forest cover, and “noise” added to see the effect on biome coverage. Derived equations consider critical slowdown (cf. the end of the ice age). Predictions based on HadCM3LC climate are being compared with major vegetation changes in HadCM3LC.

GCMs. Geostrophic balance holds in the atmosphere to a smaller scale than would be expected. It is now understood that this is owing to the nature of damping. Further progress in proving that the semi-geostrophic equations can be solved rigorously in spherical geometry supports their use as a simple model of large-scale atmospheric circulation. Recent results improve understanding of blocking climatology: blocking is likely to be more frequent with a smaller radius of deformation, which in turn is influenced by stratospheric structure (winds).

The commonly-used filter for leap-frog time-stepping had been modified recently to give much improved accuracy; the simple improvement was extended to semi-implicit schemes. Progress was also made on a new semi-implicit semi-Lagrangian dynamical core (Univ. Exeter and MetOffice collaboration), specifically on convergence of the iterative nonlinear solver.

The Hamiltonian Particle Mesh method (HPM), an alternative to Eulerian formulation of dynamical cores, discretising after an initial Hamiltonian formulation, can ensure exact transport and conservation of mass, of interest for climate simulations. Schemes have been developed for “shallow water” equations on the sphere. Tests show that HPM can represent atmospheric flows and preserves energy accurately in long-term simulations. Current experiments concern orography forcing gravity waves, moist processes and convective parameterisation due to diabatic heating.

Stratospheric processes and chemistry affect tropospheric processes and climate. Programme discussion of experiments design and Bayesian calibration helped to set up a UC London project, with Leverhulme Trust funding, to calibrate chemistry parameterizations in the NCAR Whole Atmosphere Community Climate Model.

Stochastic parameterisations and turbulence spectrum in GCMs. Processes too detailed for direct simulation in a GCM might be represented by embedding an emulator of the process model; deterministic or stochastic. A new collaboration (Univ. Aston and NCAR) is exploring emulators for superparameterisation in relation to clouds. A stochastic parameterisation of convective clouds (Univ. Reading), hitherto applied in single-column models and ensemble weather forecasting, will now be tried out for NCAR’s next climate model. Progress was made
in modelling convection out of equilibrium yet still accounting for its stochastic nature. Work is also planned to (i) improve schemes for unresolved cumulus convection by taking into account the distribution of variability in the atmospheric layer, (ii) use stochastic models to represent atmospheric dispersion through city streets (Univ. Reading and Edinburgh).

Also from the programme, NCAR are running fine- and coarse-resolution twin experiments for stochastic potential vorticity flux from sub-grid to resolved scales.

Such approaches are aided by considering how (e.g.) stratification and rotation affect energy transfers and the energy spectrum, guiding consistent derivation of sub-grid models. Fine-resolution integration of the barotropic vorticity equation was used to calculate energy tendency at each wavenumber due to wavenumbers > 96 (for example). Thereby commonly used sub-grid models were shown to dissipate energy and enstrophy over too wide a range of wavenumbers and fail to capture the backscatter of energy into small wavenumbers. Participants looked at whether other (including stochastic) subgrid models can do better.

Reduced stochastic modelling methods have a wide area of applications, with ongoing efforts to adapt to fluid dynamics, and were discussed in relation to climate modelling. The aim is to find equations of motion which sample the distribution and allow for evaluation of temporal quantities, e.g. auto-correlation functions. Two approaches were linked for first time with proof that the Nosé-Hoover-Langevin equations reduce to the Langevin equation in the limit of large stochastic variance.

**Maximum Entropy Production (MEP).** This potential general constraint was the subject of several presentations and discussion sessions. Applicability may not be clear *a priori* but surprisingly good estimates often result from use with simplified models or sub-models. Whether MEP holds for land-sea temperature contrast posed in MEP terms is being studied.

### Summary and recommendations

This programme has clearly demonstrated that many important pressing challenges in climate science require closer collaboration between mathematicians/statisticians and climate scientists. Climate science needs to embrace more advanced statistical and stochastic modelling in order to move beyond the deterministic paradigm provided by traditional *ab initio* climate models. Climate modelling issues also pose exciting challenges driving new developments in statistical science (e.g. the development of emulators capable of dealing with complex numerical models).

Some general suggestions for how this important area of interdisciplinary research can be developed include:

- More networking programmes such as this successful INI programme;
- Better recognition by climate centres of the important role of statistical and stochastic modelling – a fundamental part of the prediction system!
- More incentives for statisticians to be involved in climate science – there is a shortage of statistics skills in climate science that needs to be addressed;
- Improved funding mechanisms for joint climate-statistical research, e.g. coordinated joint funding by NERC and EPSRC. For example, a funding call for complexity mathematics in climate science would be ground-breaking and help bring in new methods such as MEP etc.

### Publications

Some papers from the August workshop will appear in a special issue of *Philosophical Transactions of the Royal Society of London A*: “Climate predictions: the influence of nonlinearity and randomness”.

TOP and SUPRAnet papers are in preparation
H. Dijkstra book "Nonlinear Climate Dynamics" nearly completed during visit at INI.


T. M. Lenton "What early warning systems are there for environmental shocks" (state of science review for UK government Foresight Project on Environmental change and migration)

T. M. Lenton, V. N. Livina et al. "Early warning of climate tipping points: a comparison of methods" for Phil Trans A special issue

T. M. Lenton, V. N. Livina et al. "Early warnings of the end of the last ice age in Greenland ice core records" (not sure where to send it yet)